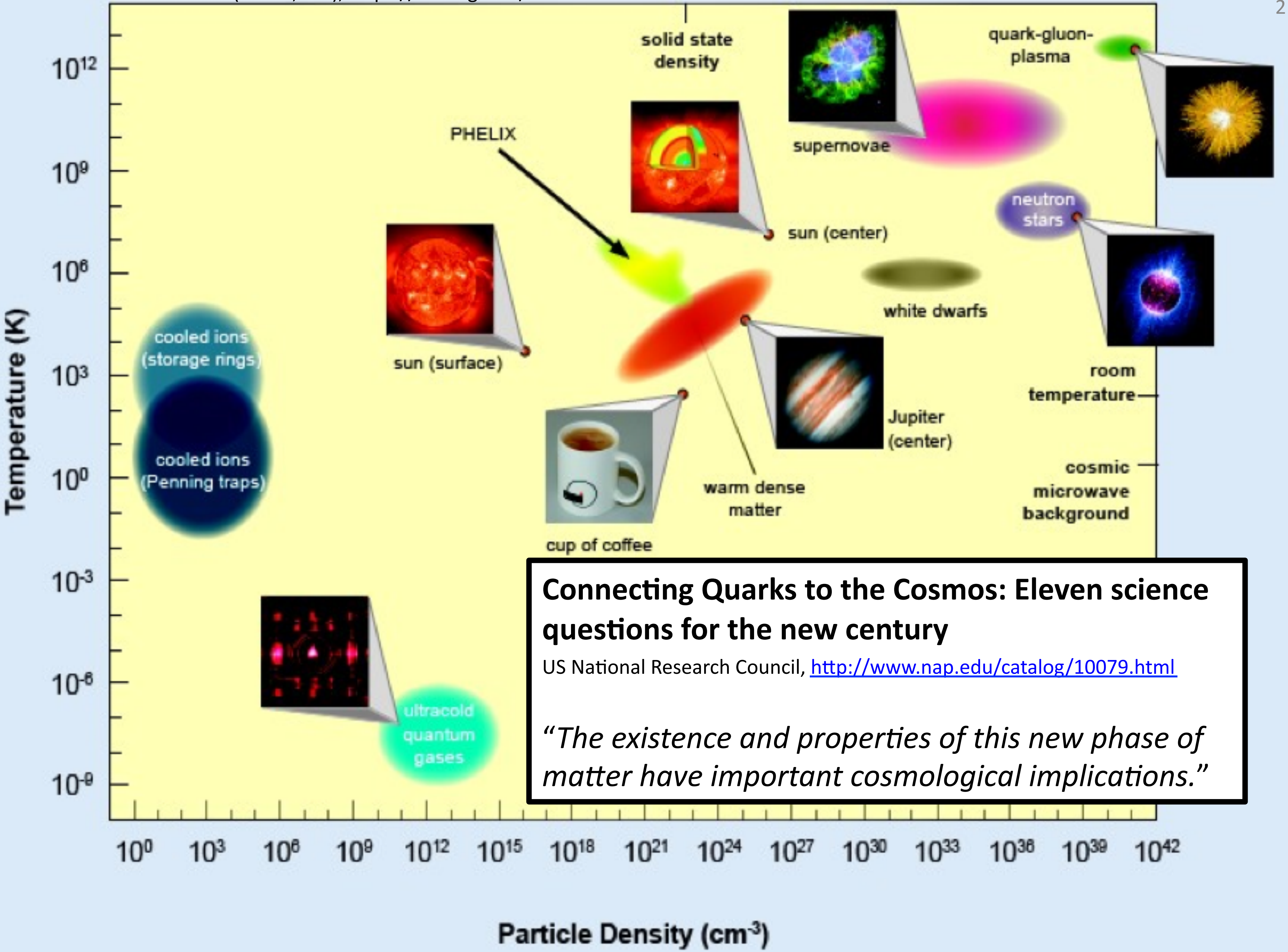
Rencontres QGP-France-15, Etretat, 12-15 octobre 2015

Quark-gluon plasma and the early Universe

arXiv:1510.04200

Philippe Rosnet
Laboratoire de Physique Corpusculaire
Université Blaise Pascal – CNRS/IN2P3

ExtreMe Matter Institute (EMMI, GSI), https://www.gsi.de/emmi



Connecting Quarks to the Cosmos: Eleven science questions for the new century

US National Research Council, http://www.nap.edu/catalog/10079.html

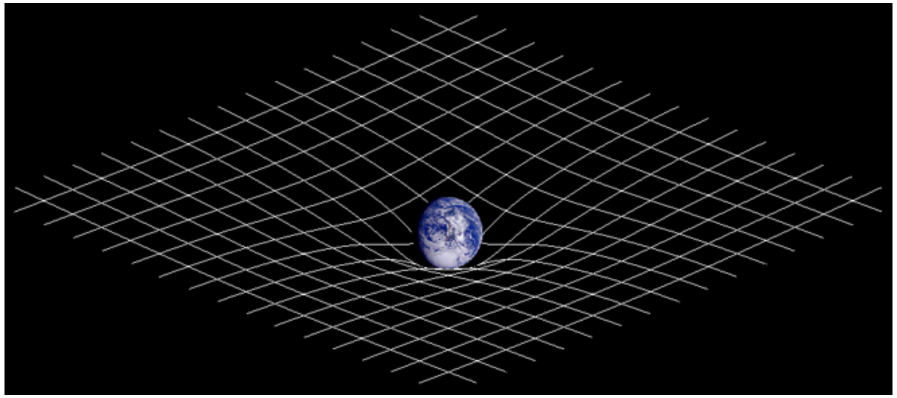
“The existence and properties of this new phase of matter have important cosmological implications.”

Basics of cosmology (1)

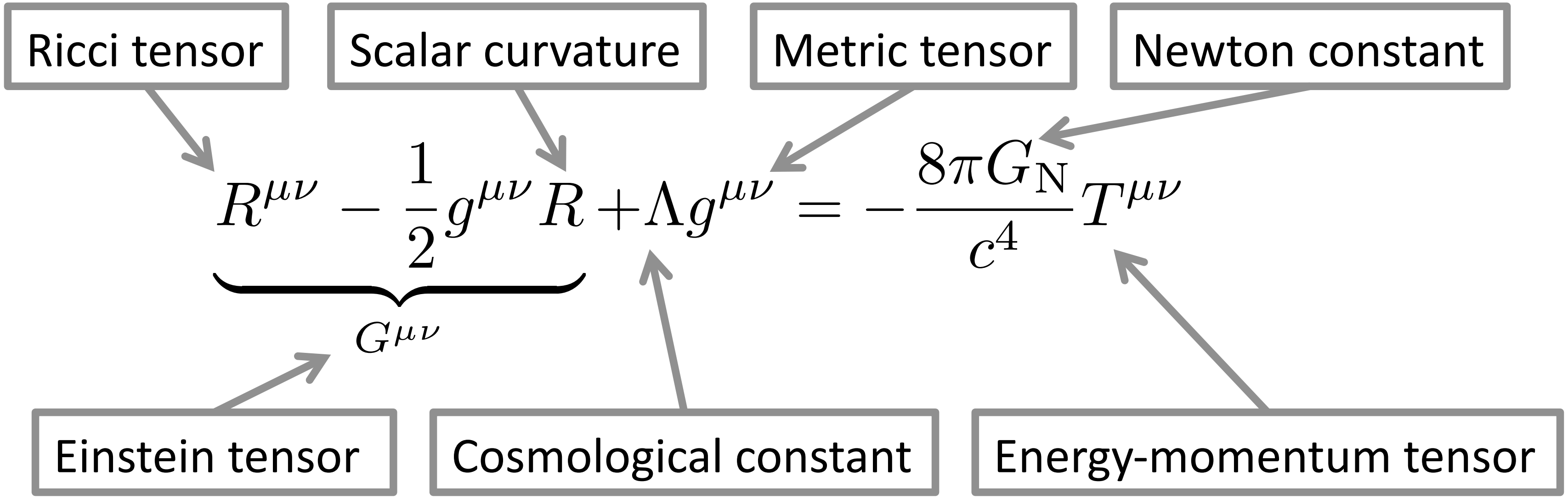
Theoretical framework = General Relativity (GR)

➔ Curved space-time

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$$



➔ Einstein's field equations



Ricci tensor

Scalar curvature

Metric tensor

Newton constant

$$\underbrace{R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R}_{G^{\mu\nu}} + \Lambda g^{\mu\nu} = -\frac{8\pi G_{\mathrm{N}}}{c^4} T^{\mu\nu}$$

Einstein tensor

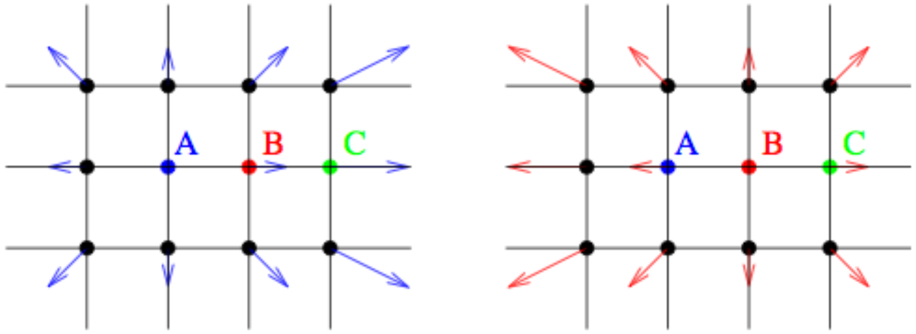
Cosmological constant

Energy-momentum tensor

"*Space tells matter how to move and matter tells space how to curve*", John Archibald Wheeler cited in C.W. Misner, K.S. Thorne and W.H. Zurek, Physics Today, April 2009, 40-46

Basics of cosmology (2)

<ins>Cosmological principle</ins>: homogeneity and isotropy of the Universe, meaning that there is no privileged point playing a particular role



<ins>Friedmann-Lemaître-Robertson-Walker (FLRW) metric</ins>:

Co-moving coordinates (t, r, θ, ϕ), *i.e.* at rest w.r.t. the entire Universe

$$ds^2 = c^2 dt^2 - a^2(t) \underbrace{\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]}_{d\sigma^2 = \text{Spatial metric}}$$

Scale factor

$$K = \begin{cases} +1 & = & \text{closed space with positive curvature} \\ 0 & = & \text{flat space with zero curvature} \\ -1 & = & \text{open space with negative curvature} \end{cases}$$

Basics of cosmology (3)

Ideal fluid: $T^{\mu\nu} = \mathrm{diag}\,(\epsilon, -P, -P, -P)$ with $\epsilon = \text{energy density} = \rho c^2$

$\rho = \text{mass density}$

$P = \text{pressure}$

Friedmann equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_\mathrm{N}}{3}\rho - \frac{Kc^2}{a^2} \quad \text{with} \quad \rho = \rho_\mathrm{m} + \rho_\mathrm{r} + \rho_\Lambda$$

Hubble parameter

Matter

Radiation

Vacuum

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G_\mathrm{N}}$$

Universe budget: $\Omega_\mathrm{m} + \Omega_\mathrm{r} + \Omega_\Lambda = 1 - \Omega_\mathrm{K}$

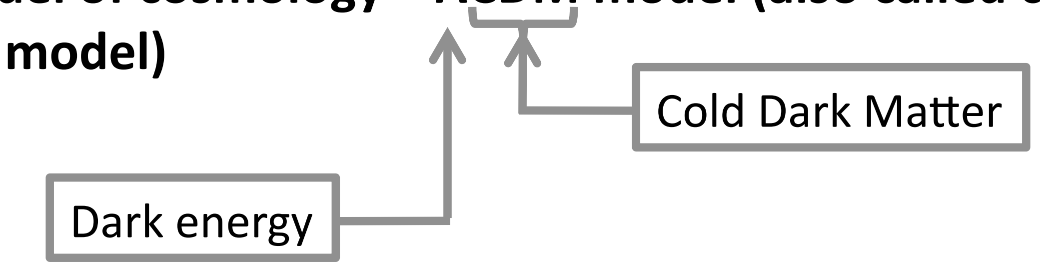
$$\text{with} \quad \Omega_\mathrm{i} = \frac{8\pi G_\mathrm{N}}{3H^2}\rho_\mathrm{i} \quad \text{and} \quad \Omega_\mathrm{K} = \frac{Kc^2}{H^2 a^2}$$

Basics of cosmology (4)

<u>Equation of state</u>: $P = w\,\epsilon \quad \text{with} \quad \frac{d\epsilon}{dt} = -3\sqrt{\frac{8\pi G_{\rm N}\epsilon}{3}}(\epsilon + P)$

- $w = 0$ for non relativistic matter (no pressure)
  $a(t) \propto t^{2/3}$ and $H(t) = \frac{2}{3t}$
- $w = \frac{1}{3}$ for radiation or ultra-relativitic (massless) matter
  $a(t) \propto t^{1/2}$ and $H(t) = \frac{1}{2t}$
- $w = -1$ for the cosmological constant
  $a(t) \propto \exp\left(\sqrt{\frac{\Lambda c^2}{3}}t\right)$ and $H = \sqrt{\frac{\Lambda c^2}{3}}$

Standard model of cosmology = ΛCDM model (also called cosmological concordance model)



An expanding Universe

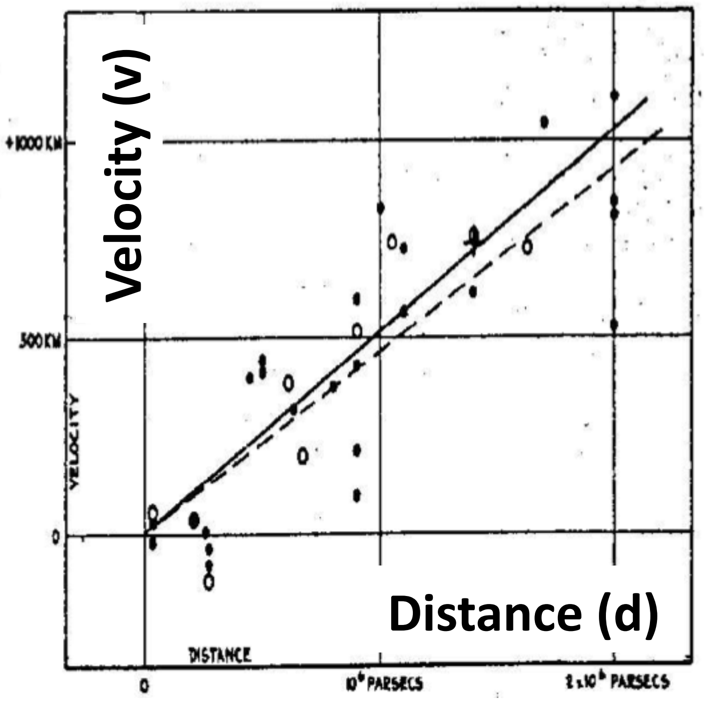
- Distance to a galaxy at t_0 (today)

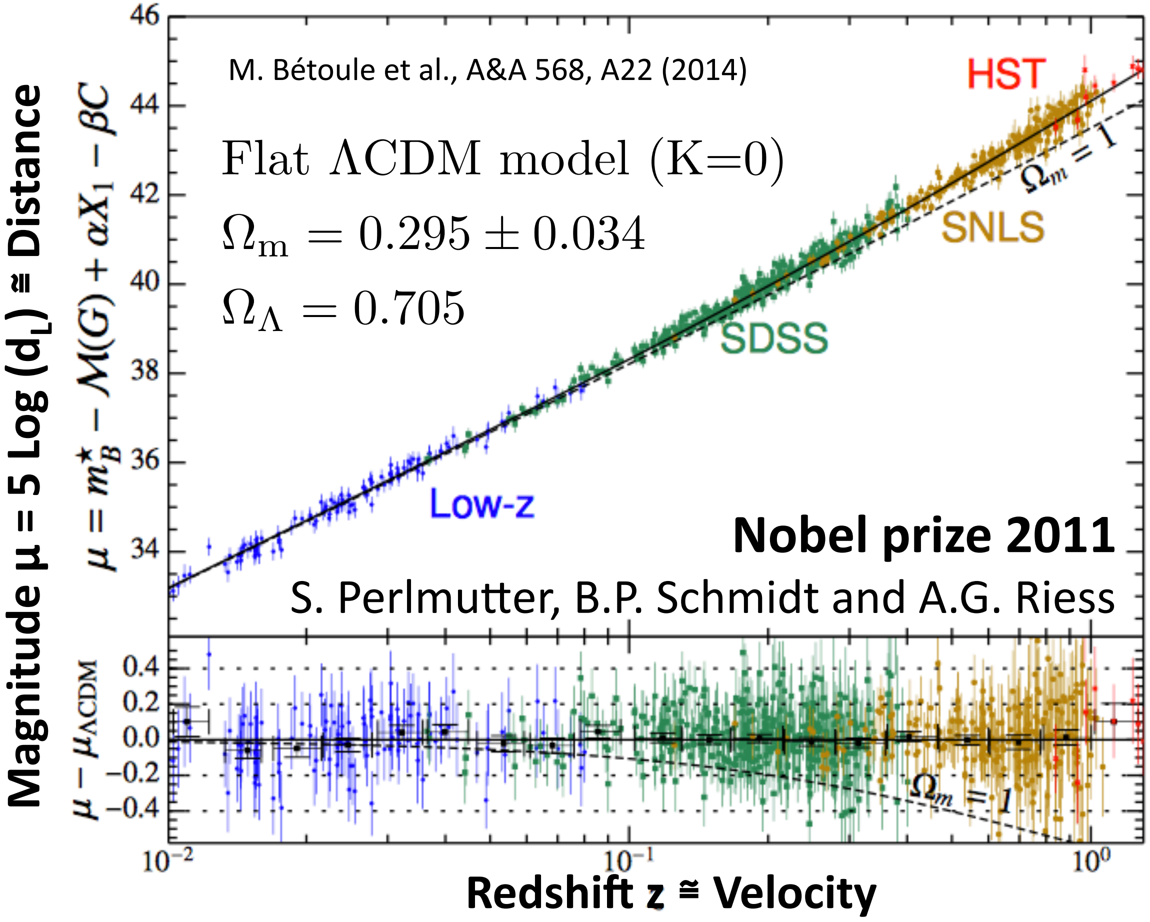
$$d = a_0 r \quad \text{with } r \text{ representing the fixed co-moving coordinate}$$

- Hubble law (for near galaxies): $\dot{a} = Ha \quad \Rightarrow \quad v = H_0 d$

- Hubble diagram

Diagram published by Hubble in 1929,
Cited in J. Lesgourgues, *An overview of cosmology*,
CERN-2005-013





M. Bétoule et al., A&A 568, A22 (2014)

Flat ΛCDM model (K=0)

$\Omega_{\text{m}} = 0.295 \pm 0.034$

$\Omega_\Lambda = 0.705$

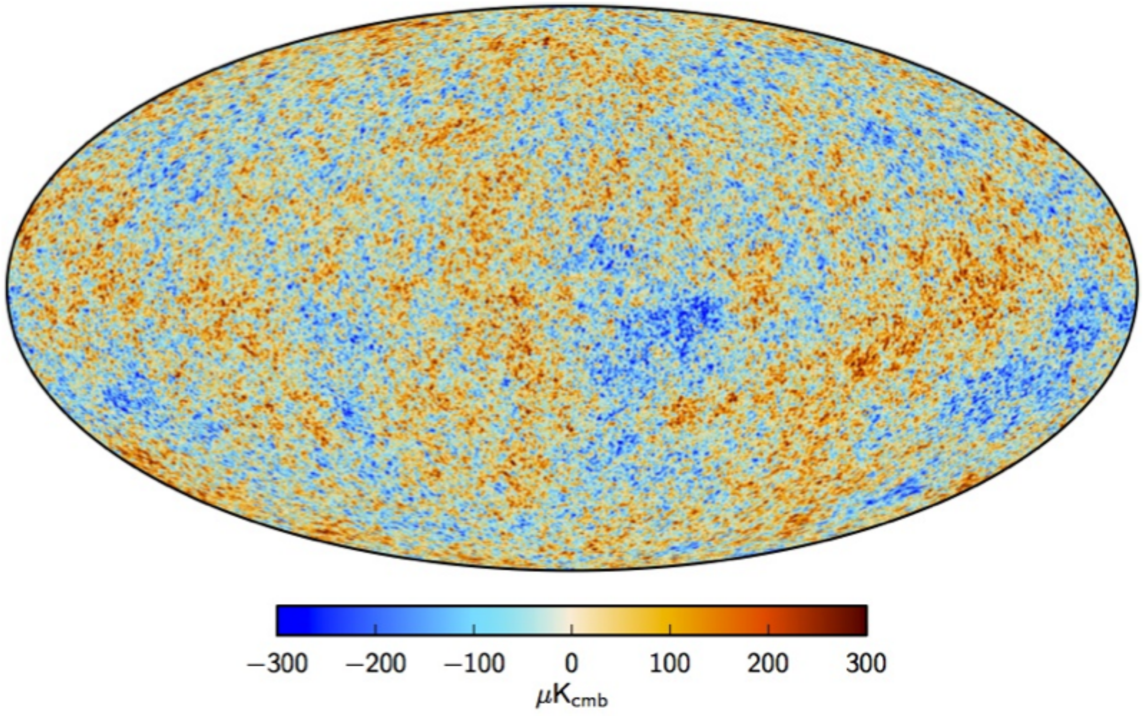
Nobel prize 2011

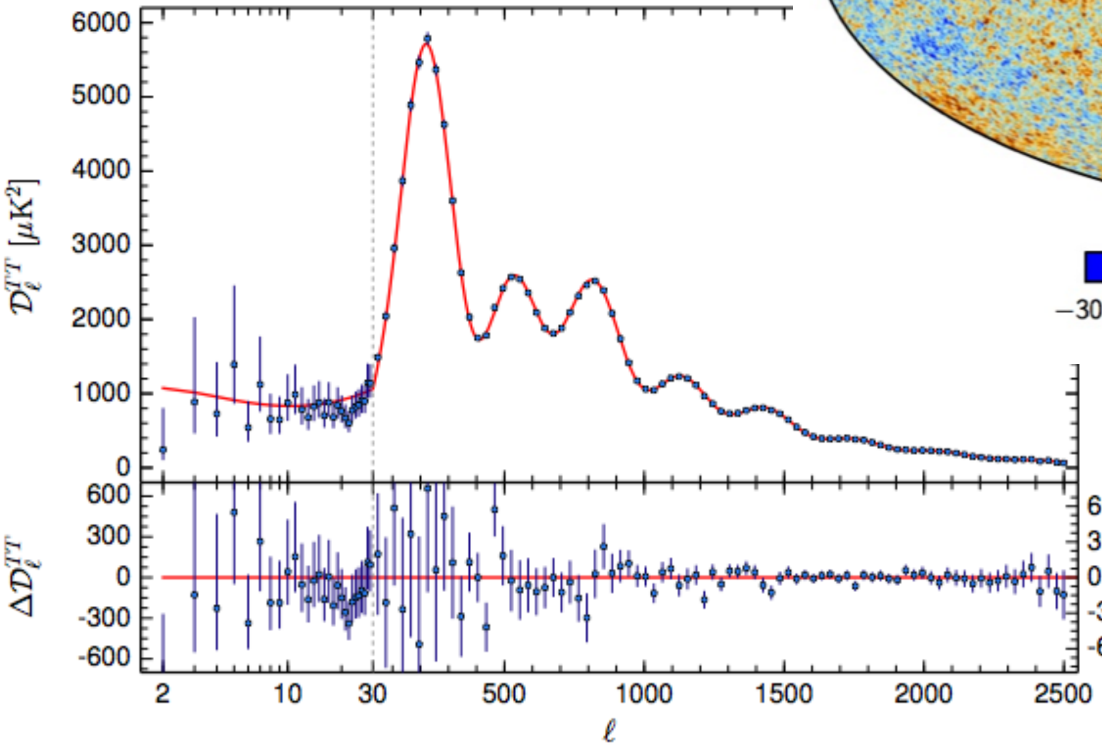
S. Perlmutter, B.P. Schmidt and A.G. Riess

A flat Universe

Planck Collaboration, arXiv:1502.01582 [astro-ph]

Cosmic Microwave Background (CMB) temperature map





Temperature angular power spectrum (acoustic peaks) and ΛCDM model fit

ΛCDM model fit result (Planck + external data)

Planck Collaboration, arXiv:1502.01589 [astro-ph]

$$\Omega_{\rm K} = 0.0008 \pm^{0.0040}_{0.0039} \quad @95\%{\rm CL}$$

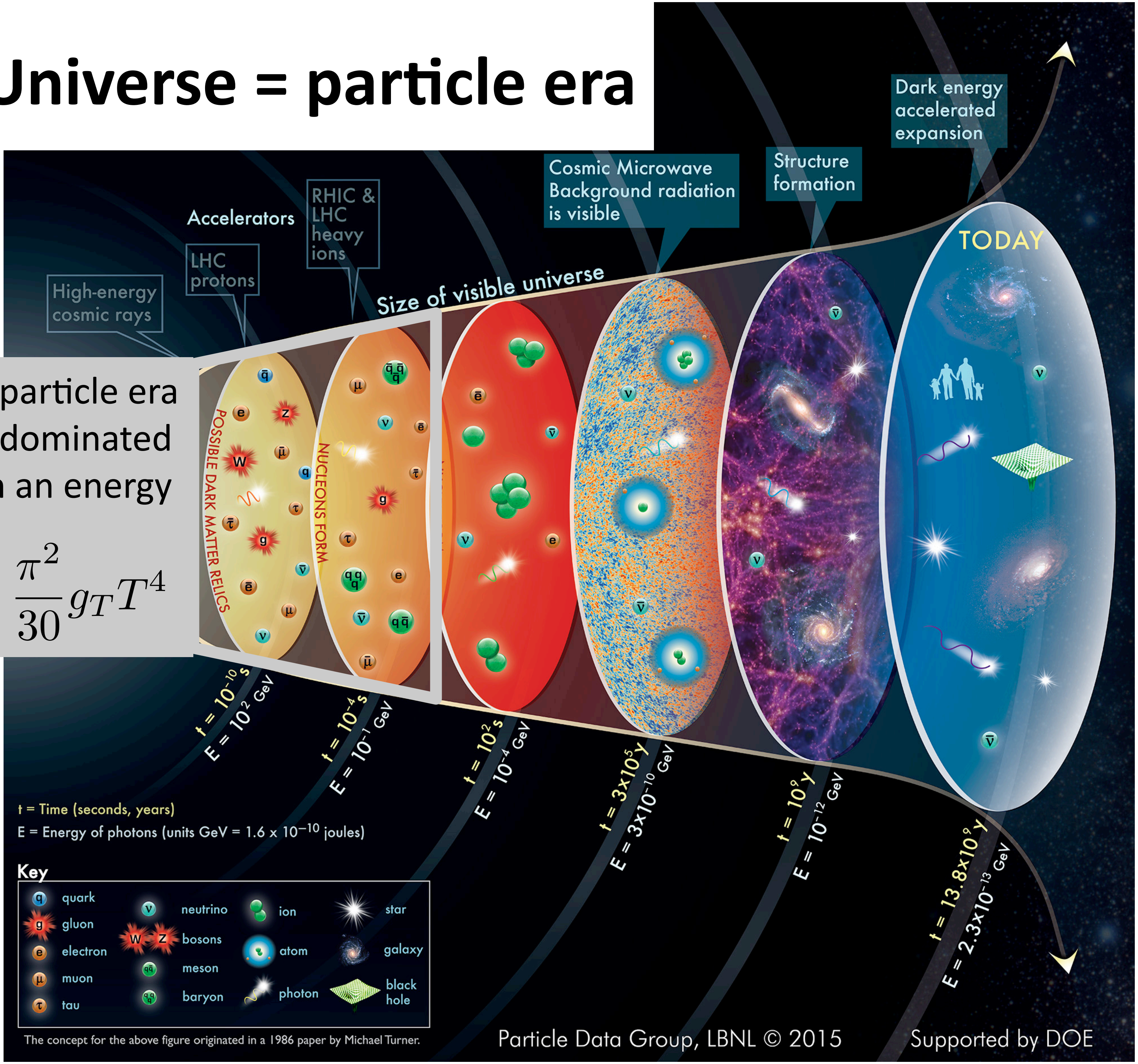
$$\Omega_{\rm m} = 0.3089 \pm 0.0062 \quad @68\%{\rm CL}$$

$$\Omega_{\Lambda} = 0.6911 \pm 0.0062 \quad @68\%{\rm CL}$$

Early Universe = particle era

Ultra-relativistic particle era means Universe dominated by radiation with an energy density

$$\epsilon_{\rm rad} = \frac{\pi^2}{30} g_T T^4$$



The concept for the above figure originated in a 1986 paper by Michael Turner.

Particle Data Group, LBNL © 2015

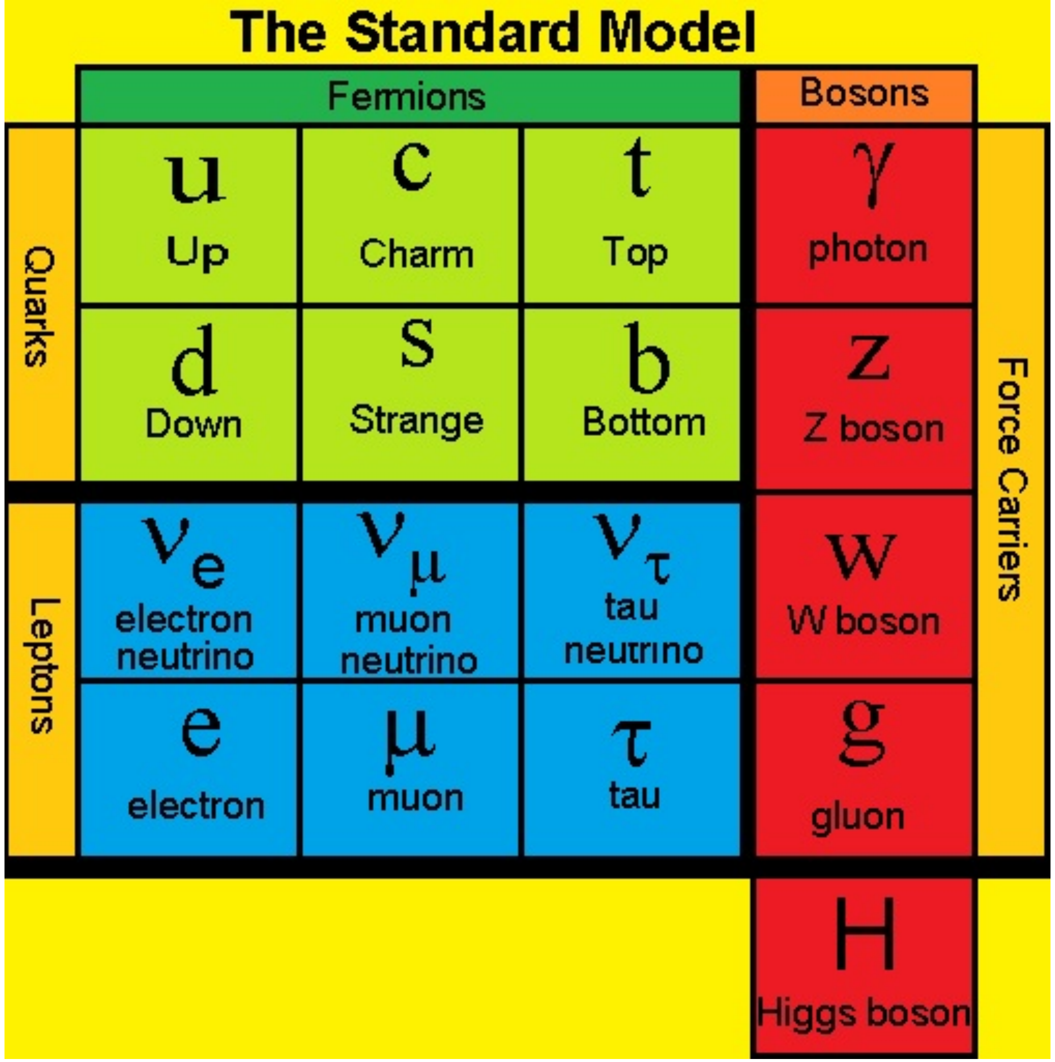
Supported by DOE

The Standard Model of particle physics

Fermions: spin ½

Quarks: 3 colours

Neutrinos: only left-handed state



The Standard Model

	Fermions			Bosons	
Quarks	u Up	c Charm	t Top	γ photon	Force Carriers
	d Down	s Strange	b Bottom	Z Z boson	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
	e electron	μ muon	τ tau	g gluon	
				H Higgs boson	

Gauge bosons: spin 1

Gluons: 8 colour states

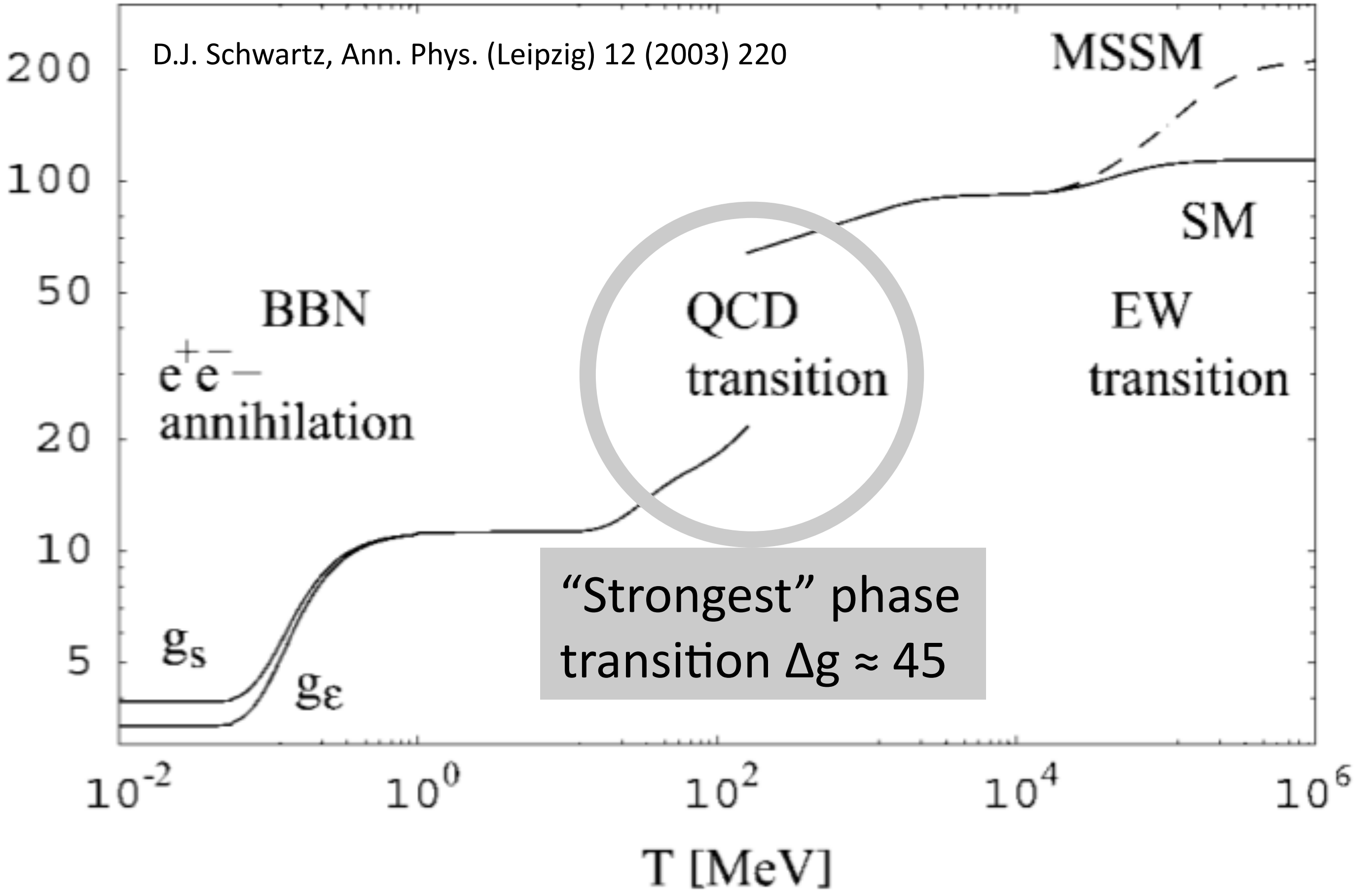
Higgs boson: spin 0

+ anti-fermions

Degrees of freedom in Standard Model era

Counting degrees of freedom in the Standard Model with $T >> m_{top} = 173$ GeV

- $g_{boson} = 4_{Higgs} + 4_{EW\ boson} \times 2_{massless\ spin\ 1} + 8_{gluon} \times 2_{massless\ spin\ 1} = 28$
- $g_{fermion} = 2_{(anti\text{-})particle} \times 3_{family} \times (2_{quark} \times 3_{color} \times 2_{spin\ ½} + 1_{l-} \times 2_{spin\ ½} + 1_{\nu} \times 1_{Left}) = 90$
- $g_{SM} = g_{boson} + (⅞)_{massless} \times g_{fermion} \approx 107$



D.J. Schwartz, Ann. Phys. (Leipzig) 12 (2003) 220

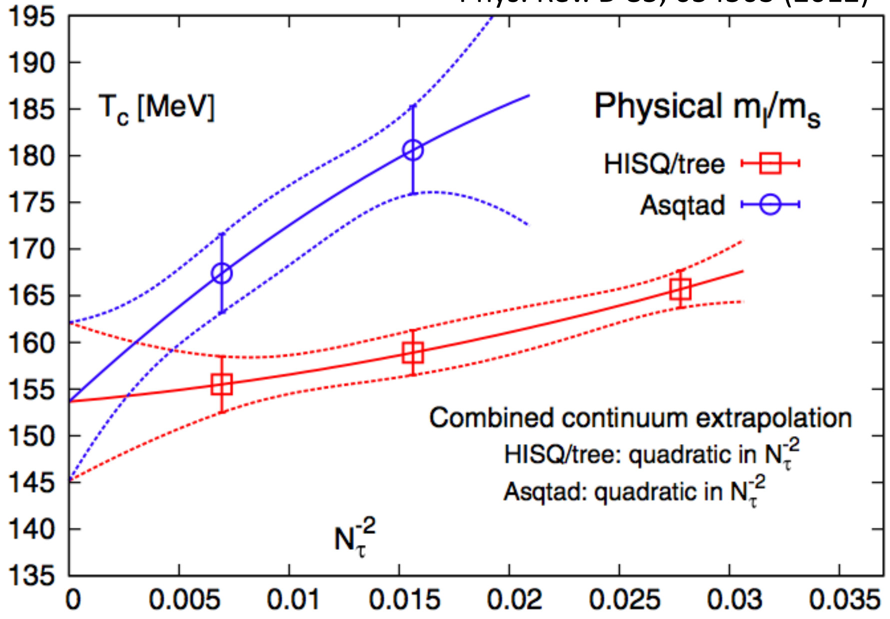
"Strongest" phase transition Δg ≈ 45

QCD transition: last lattice results

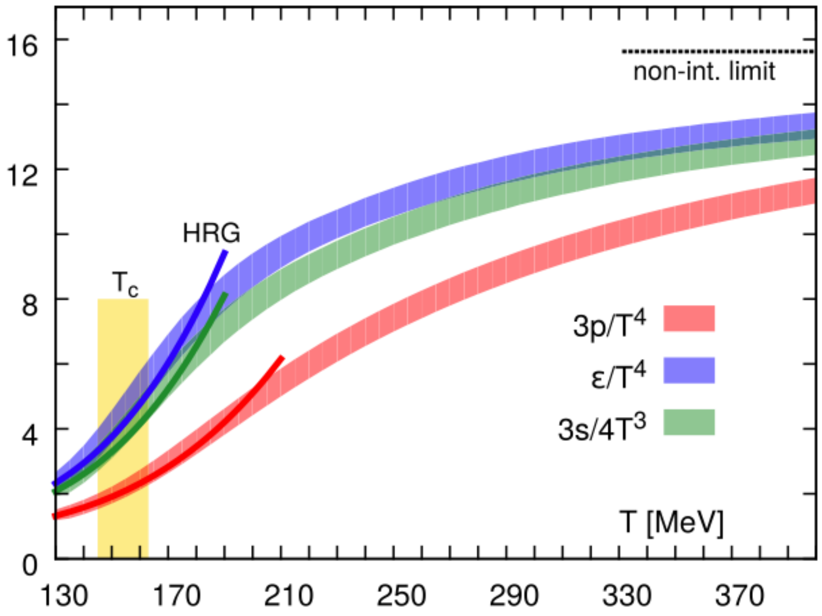
Transition temperature at zero net baryon density with lattice QCD calculation extrapolated to continuum limit

T_{QCD} = 154 ± 9 MeV
≈ 1.8 × 10^{12} K

Phys. Rev. D 85, 054503 (2012)



Phys. Rev. D 90, 094503 (2014)



Corresponding energy density at transition

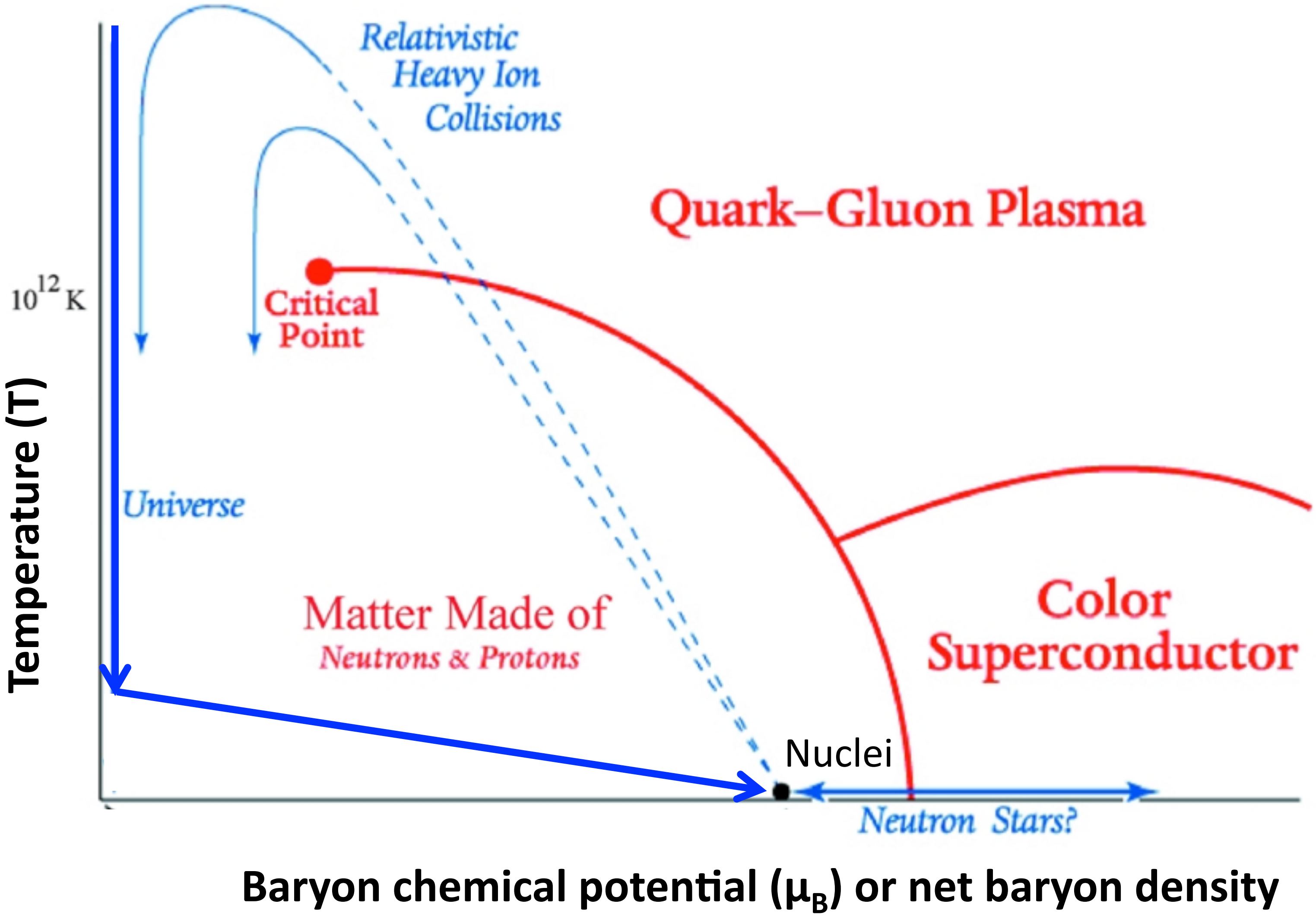
ϵ_c = (0.18–0.5) GeV/fm^3

$$\frac{3P}{T^4} \neq \frac{\epsilon}{T^4}$$

QCD equation of state (EoS) ≠ Radiation EoS

QCD phase diagram: Universe path

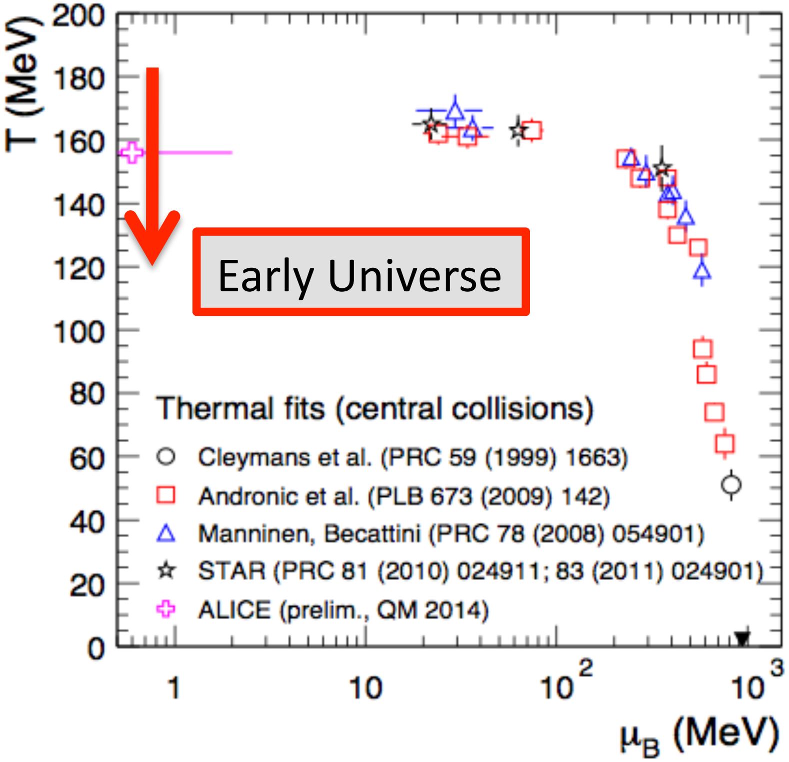
US National Research Council, http://www.nap.edu/catalog/10079.html
modified following P. Minkowski and S. Kabana, J. Phys. G: Nucl. Phys. 28 (2002) 2063-2067



Measuring the transition temperature

Statistical hadronization of quarks and gluons in hadrons

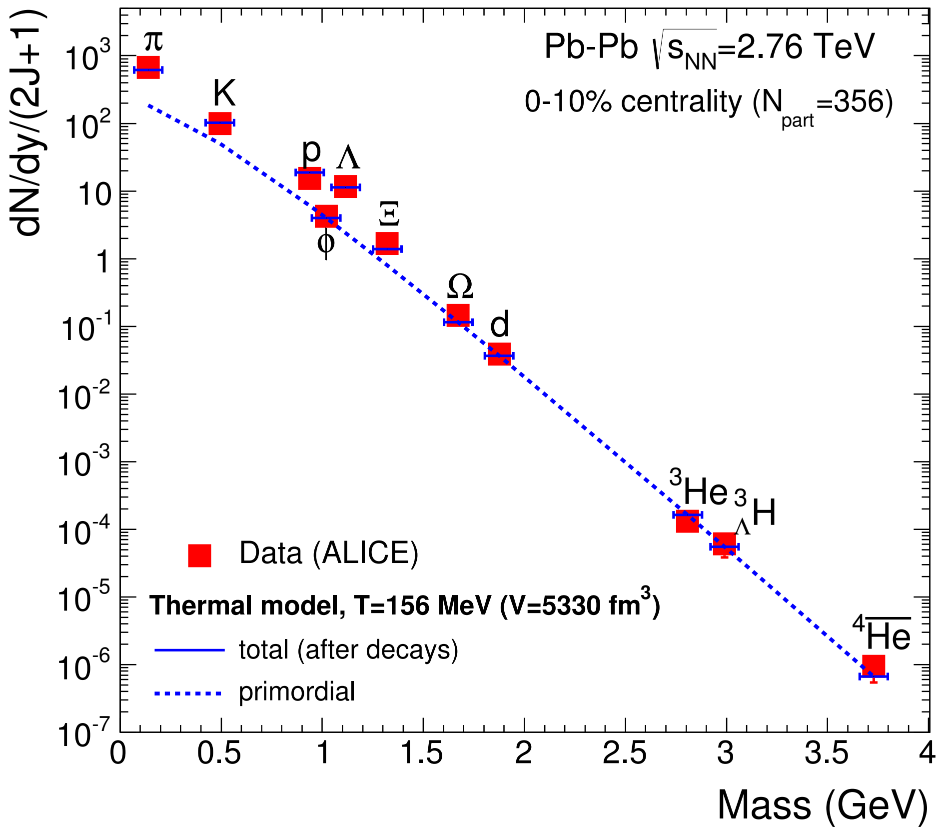
- Freeze-out temperature (≈ transition temperature): T_{fo} = 156 MeV
- Baryon chemical potential at freeze-out: μ_B = 0



Early Universe

A. Andronic, Int. J. Mod. Phys. A 2014.29

A. Andronic, Int. J. Mod. Phys. A 2014.29



QCD transition explored at LHC is very closed to early Universe one

Radiation EoS

$$\begin{cases} \epsilon = \frac{\pi^2}{30} g_T T^4 \\ P = \frac{\pi^2}{90} g_T T^4 \end{cases} \quad \text{with} \quad g_T = \begin{cases} g_1 = g_{\mathrm{EW}} + g_{\mathrm{QGP}} & \text{for} \quad T \gtrsim T_{\mathrm{QCD}} \\ g_2 = g_{\mathrm{EW}} + g_{\mathrm{HG}} & \text{for} \quad T \lesssim T_{\mathrm{QCD}} \end{cases}$$

$$\text{and} \quad g_{\mathrm{EW}} = 14.25$$

Friedmann's equation gives $t_{[\mathrm{s}]} = \frac{1}{2H} = \frac{2.42}{\sqrt{g_T}(T_{[\mathrm{MeV}]})^2}$

QCD phase transition timing assuming a temperature transition T_{QCD} = 154 MeV

	$g_{\text{photon (+ gluons or pions)}}$	$g_{\text{leptons (+ 3 quarks)}}$	g_{tot}	t
Transition starting point	18	50	$g_1 \approx 62$	13 μs
Transition end point	5	14	$g_2 \approx 17$	25 μs

$$\boxed{\Delta t_{QCD} \approx 12\ \mu s}$$

Bag EoS

K. Yagi, T. Hatsuda and Y. Miake, Camb. Monogr. Part. Phys. Nucl. Phys. 23 (2005)

Bag model: free massless quarks and gluons bounded by a negative pressure, the bag constant

$$B = (g_{\mathrm{QGP}} - g_{\mathrm{HG}})\frac{\pi^2}{90}T_{\mathrm{QCD}}^4$$

$$T \gtrsim T_{\mathrm{QCD}} \qquad \begin{cases} \epsilon = \dfrac{\pi^2}{30} g_1 T^4 + B \\ P = \dfrac{\pi^2}{90} g_1 T^4 - B \end{cases}$$

$$T = T_{\mathrm{QCD}} \qquad \begin{cases} \epsilon = [1 - f(t)]\left[\dfrac{\pi^2}{30} g_1 T_{\mathrm{QCD}}^4 + B\right] + f(t)\dfrac{\pi^2}{30} g_2 T_{\mathrm{QCD}}^4 \\ \text{with} \quad 0 \leq f(t) \leq 1 \\ P = \dfrac{\pi^2}{90} g_1 T_{\mathrm{QCD}}^4 - B = \dfrac{\pi^2}{90} g_2 T_{\mathrm{QCD}}^4 \end{cases}$$

$$T \lesssim T_{\mathrm{QCD}} \qquad \begin{cases} \epsilon = \dfrac{\pi^2}{30} g_2 T^4 \\ P = \dfrac{\pi^2}{90} g_2 T^4 \end{cases}$$

Cross-over EoS

Realistic EoS W. Florkowski, Nucl. Phys. A 853 (2011) 173-188

- using lattice prediction constrained by heavy-ion RHIC data
- numerical solution $$\frac{d\epsilon}{dt} = -3\sqrt{\frac{8\pi G_{\rm N}\epsilon}{3}}(\epsilon + P)$$

with thermodynamic relations

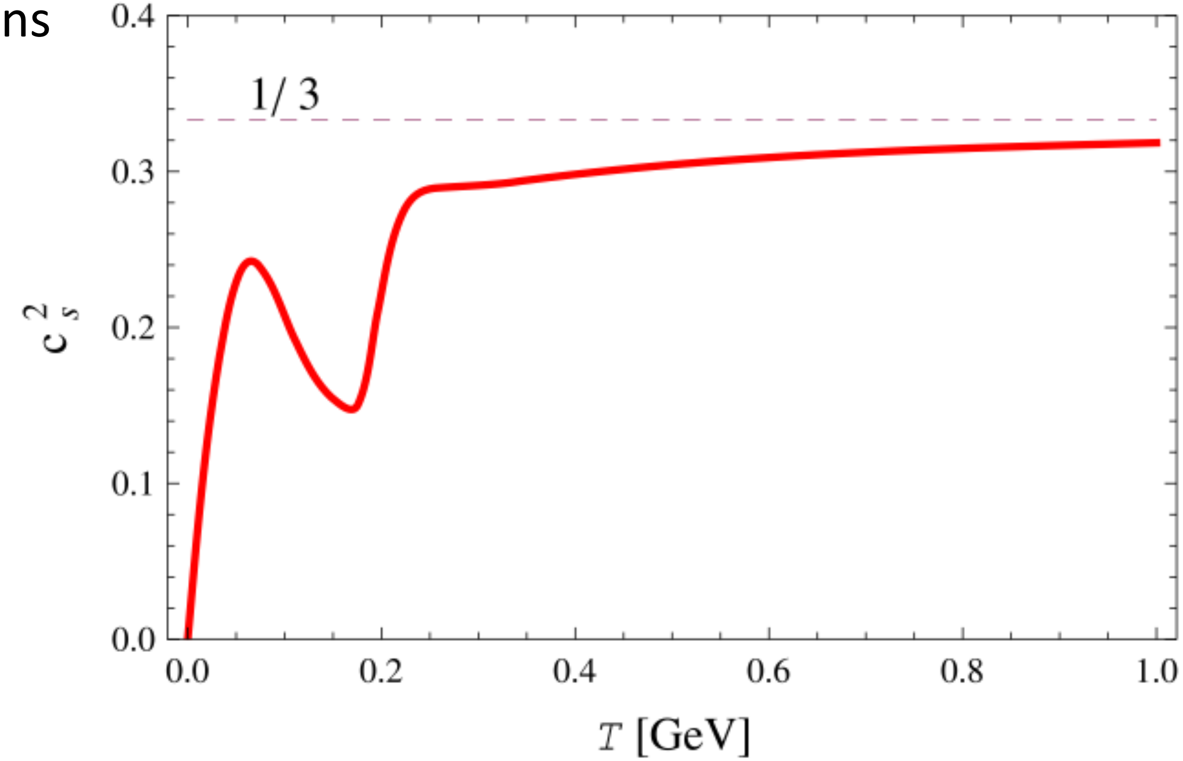
$$\epsilon + P = T\,s$$

$$d\epsilon = T\,ds$$

$$dP = s\,dT$$

and the sound velocity

$$c_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{s}{T}\frac{\partial T}{\partial s}$$



Temperature evolution

Left: first order QCD transition

- radiation EoS
- bag EoS

with

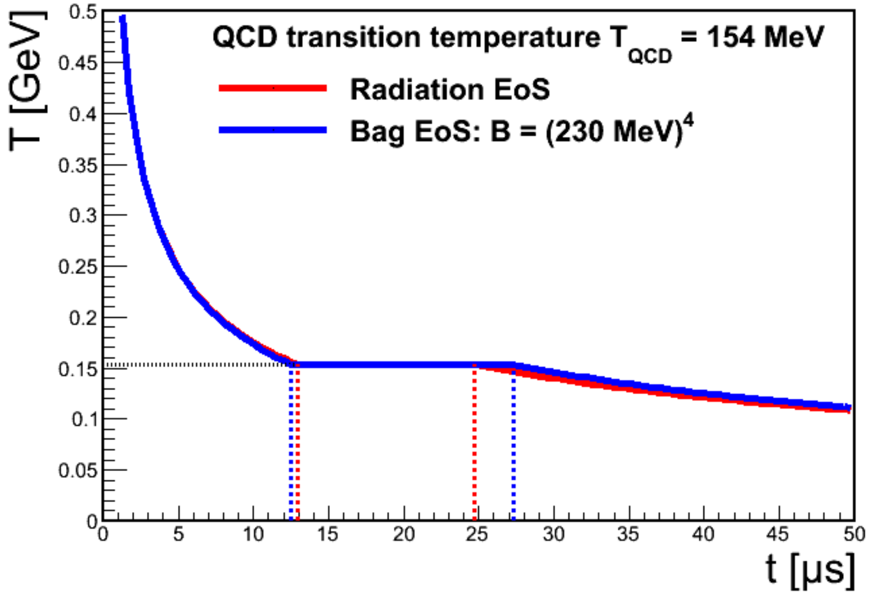
- T_{QCD} = 154 MeV
- g_1 = 61.75 (3 quark flavours)
- g_2 = 17.25

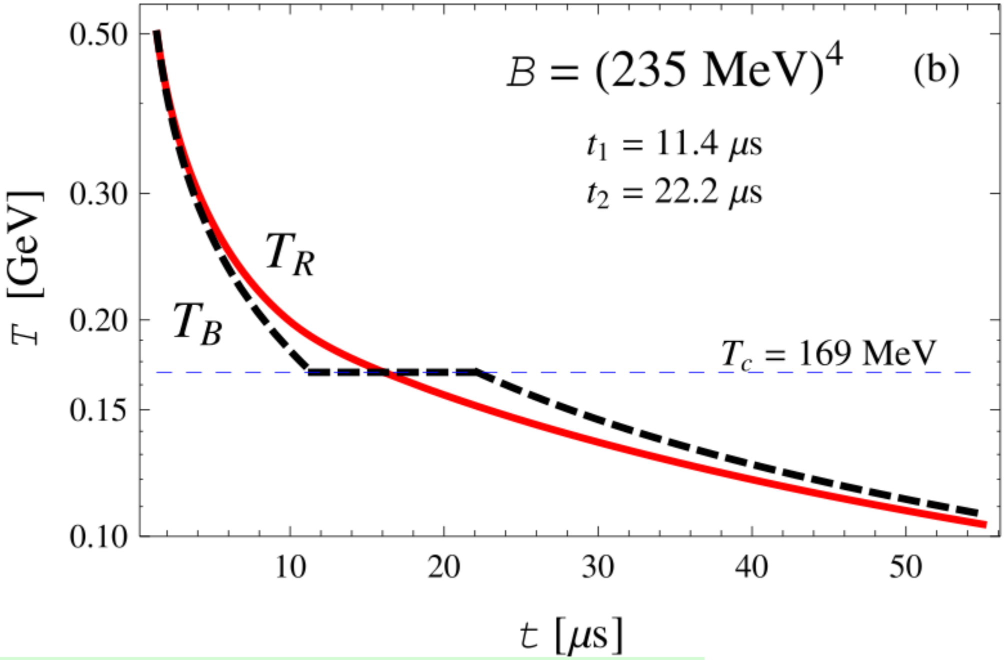
Right:

- bag EoS
- *realistic* EoS

with

- T_{QCD} = 169 MeV
- g_1 = 51.25 (2 quark flavours)
- g_2 = 17.25





Temperature plateau not present in QCD cross-over transition

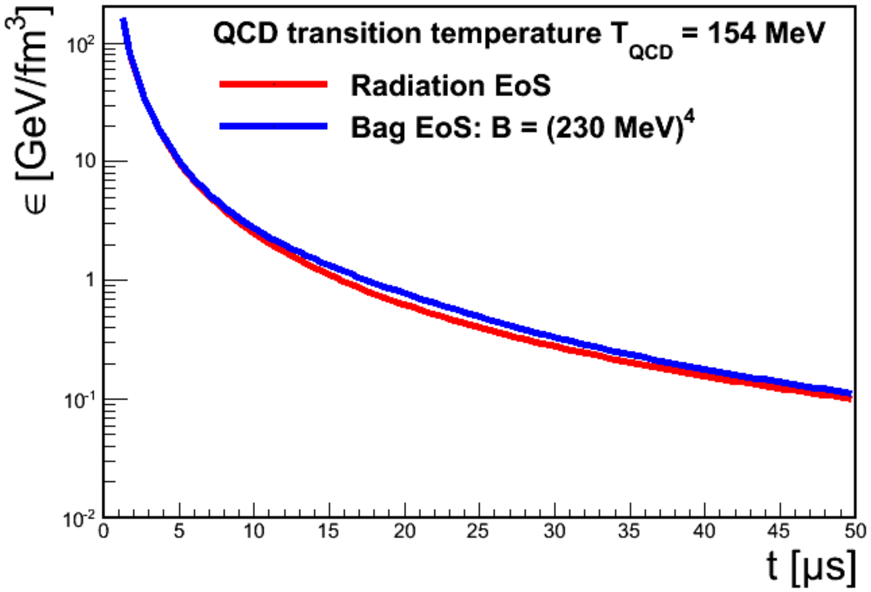
Energy density evolution

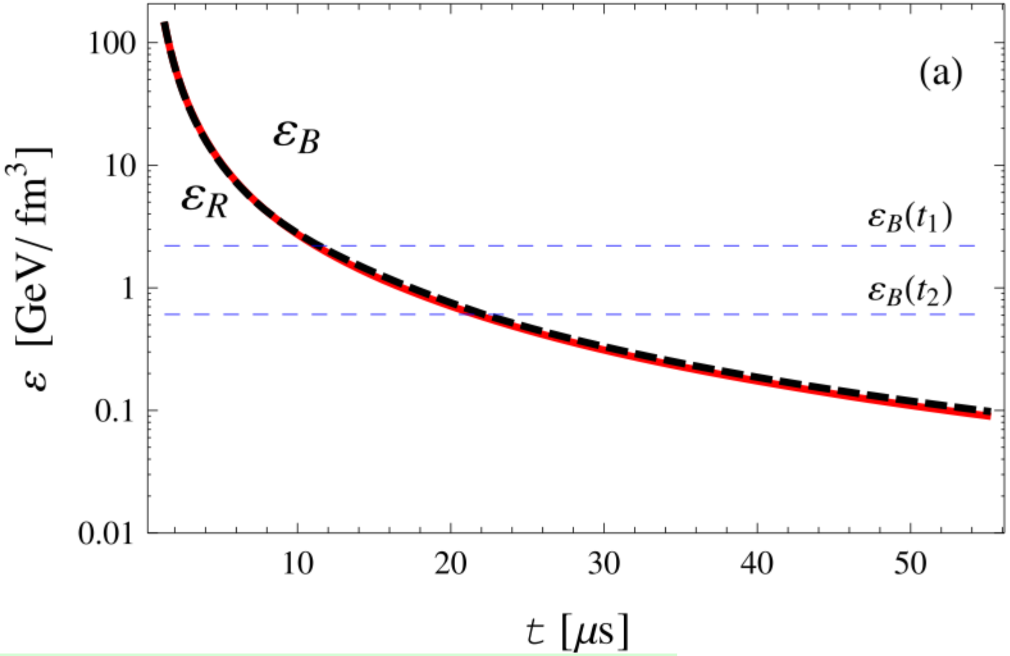
Left: first order QCD transition
- radiation EoS
- bag EoS

with
- T_{QCD} = 154 MeV
- g_1 = 61.75 (3 quark flavours)
- g_2 = 17.25

Right:
- bag EoS
- *realistic* EoS

with
- T_{QCD} = 169 MeV
- g_1 = 51.25 (2 quark flavours)
- g_2 = 17.25





Smooth evolution of energy density with all approaches

Pressure evolution

Left: first order QCD transition

- radiation EoS
- bag EoS

with

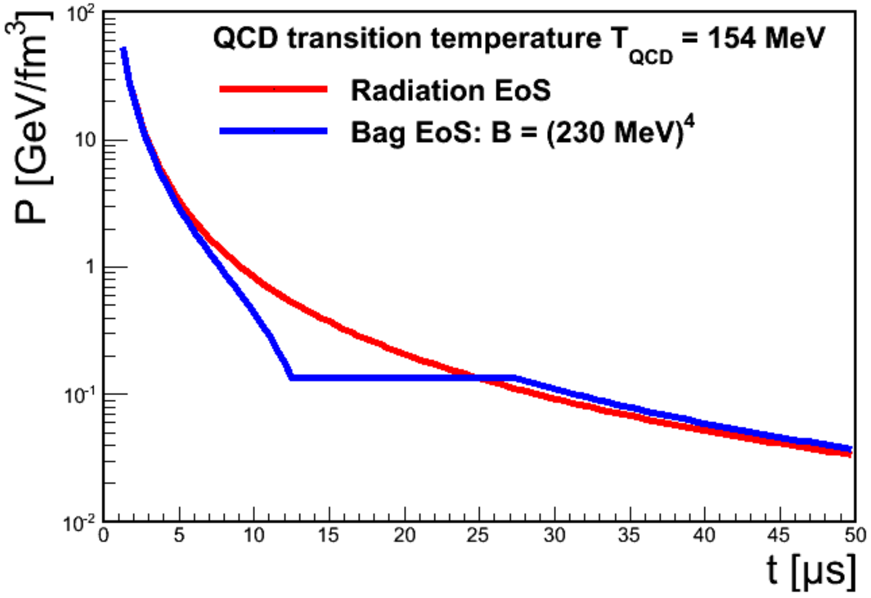
- T_{QCD} = 154 MeV
- g_1 = 61.75 (3 quark flavours)
- g_2 = 17.25

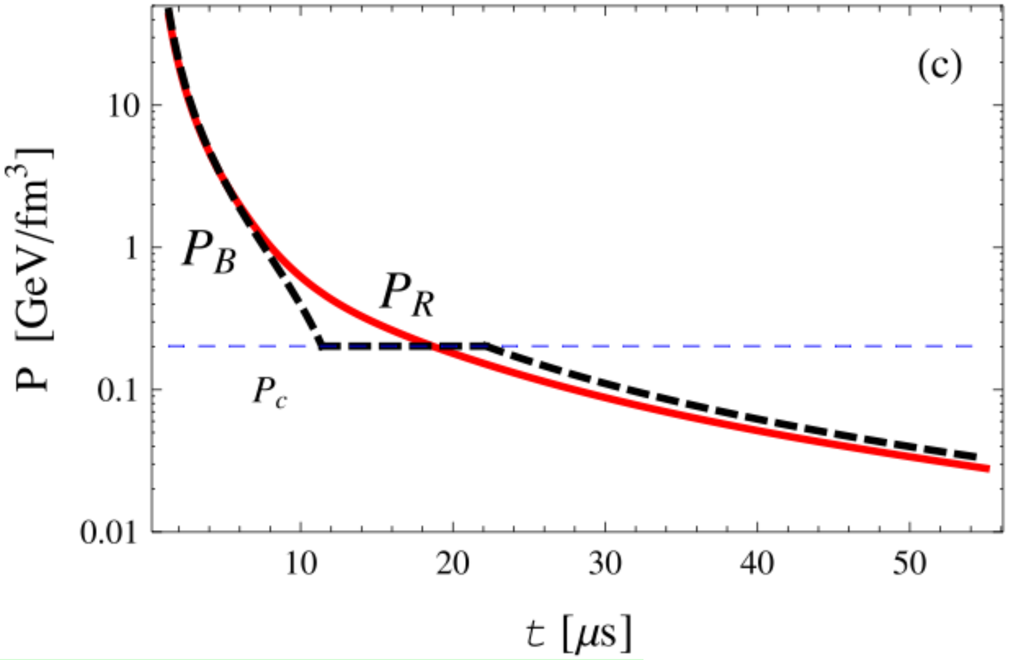
Right:

- bag EoS
- *realistic* EoS

with

- T_{QCD} = 169 MeV
- g_1 = 51.25 (2 quark flavours)
- g_2 = 17.25





Bag EoS presents a pressure plateau at QCD transition

Scale factor

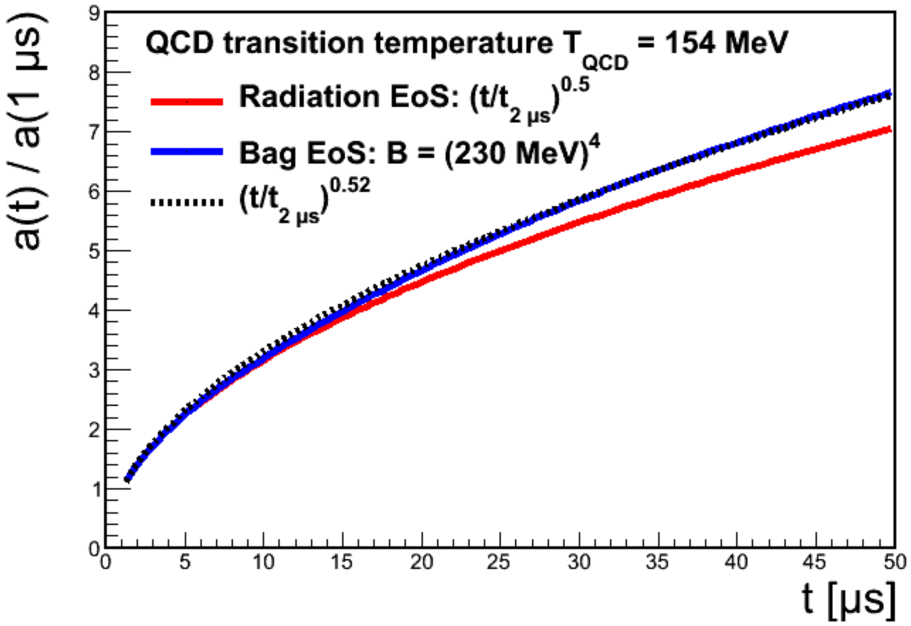
$$a(t) = a(t_0)\exp\left[\int_{t_0}^{t}\sqrt{\frac{8\pi G_{\mathrm{N}}\epsilon(t')}{3c^3}}dt'\right]$$

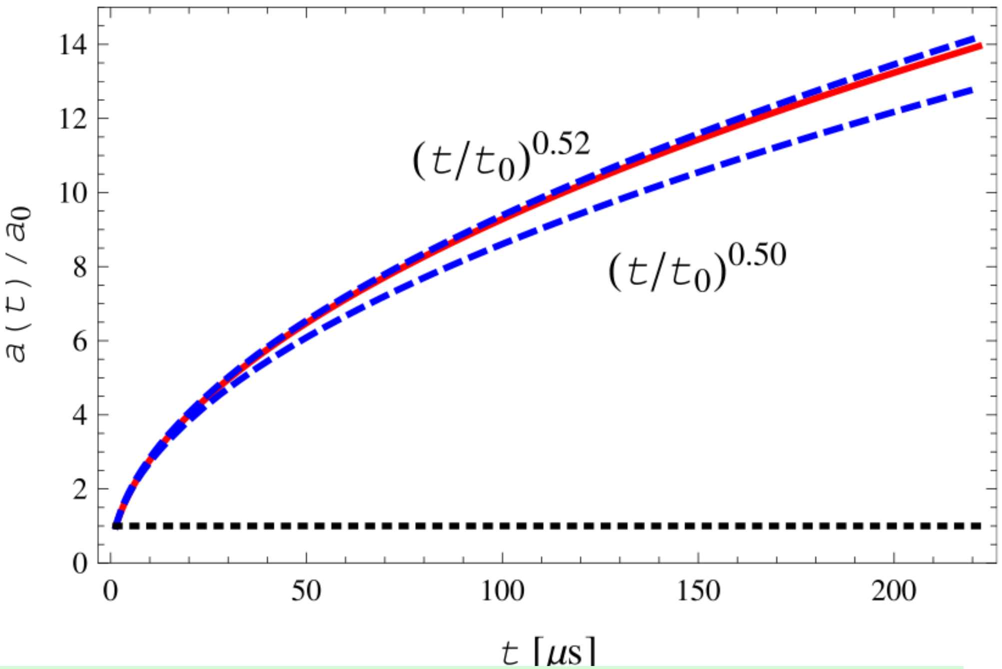
Left: first order QCD transition
- radiation EoS
- bag EoS

with
- T_{QCD} = 154 MeV
- g_1 = 61.75 (3 quark flavours)
- g_2 = 17.25

Right:
- bag EoS
- *realistic* EoS

with
- T_{QCD} = 169 MeV
- g_1 = 51.25 (2 quark flavours)
- g_2 = 17.25





Scale factor evolution with QCD EoS deviates slightly from radiation expectation (√t)

Impact on cosmological inhomogeneities

Density perturbation $\epsilon(\mathbf{x},t)=\epsilon_0(t)+\delta\epsilon(\mathbf{x},t)$

Density contrast $\delta=\frac{\delta\epsilon}{\epsilon}$

Conformal time $\eta=\int\frac{dt}{a(t)}$

- Bag EoS (1^{st} order phase transition)

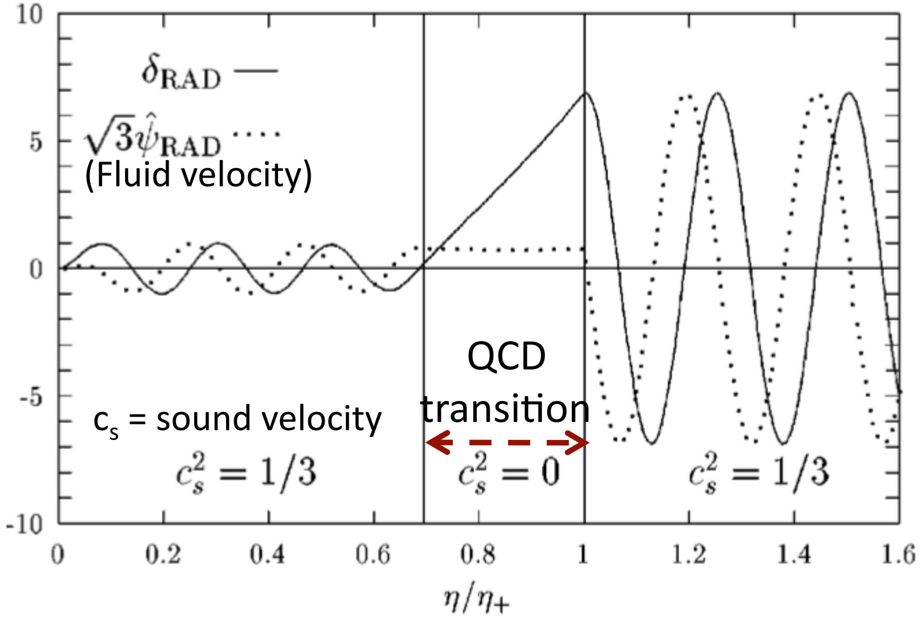
➔ amplification of cosmological inhomogeneities

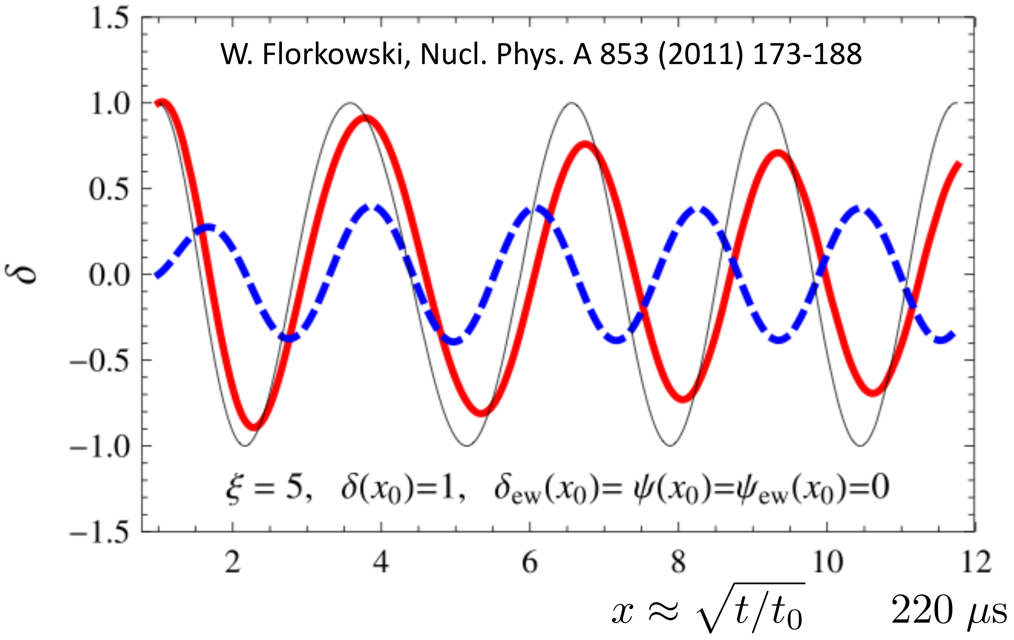
- Cross-over EoS

▬ QCD attenuation (-30%)
➔ opposite trend w.r.t.
1^{st} order QCD transition

╍ EW oscillations triggered
by QCD phase transition

C. Schmid, D.J. Schwarz and P. Widerin, Phys. Rev. D 59 (1999) 043517




W. Florkowski, Nucl. Phys. A 853 (2011) 173-188

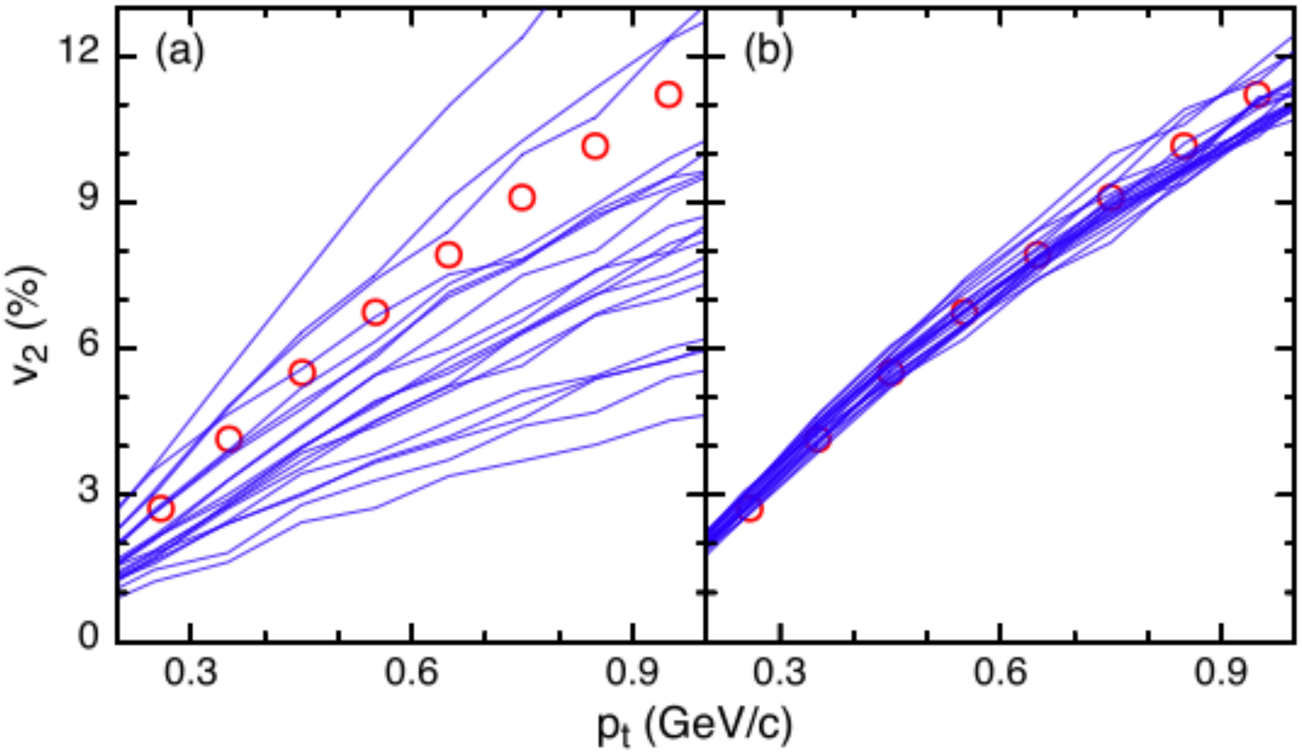
Inhomogeneities in the early Universe very dependent on the type of QCD EoS

QCD EoS: experimental constraints (1)

S. Pratt et al., Phys. Rev. Lett. 114, 022301 (2015)

Statistical analysis using Markov-chain Monte Carlo calculation

- to constrain 14 independent model parameters = (4 for stress-enegy tensor + 1 for flow + 1 for viscosity) for each RHIC and LHC energy + **2 for EoS**
- w.r.t. 30 observables (15 RHIC + 15 LHC) = (mean transverse momentum of pions, kaons and protons + pion yield + 3 femtoscopic sizes) for each 0-5% and 20-30% centrality classes and elliptic flow for 20-30% class



Pion v_2 from ALCE 20-30% centrality class

(a) Model prediction from random parameters in pre-defined parameter space

(b) Model prediction in constrained parameter space

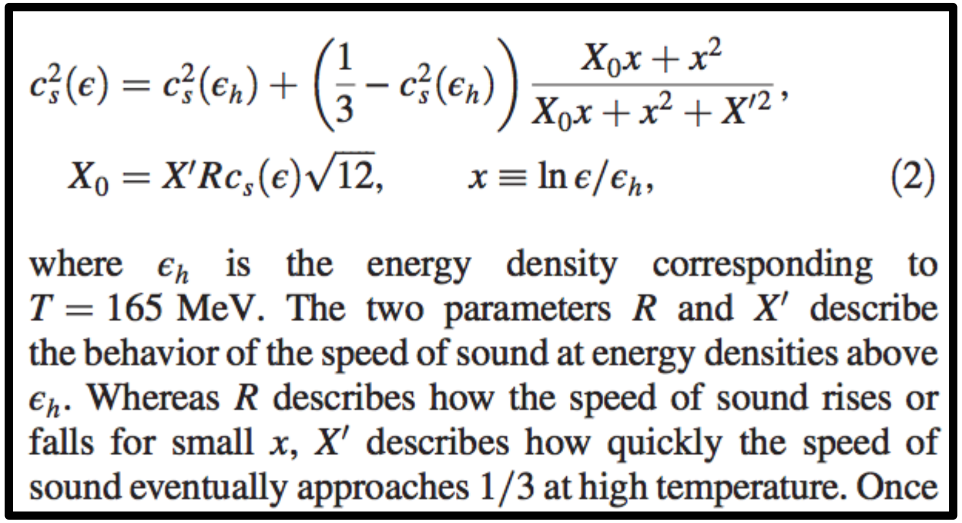
QCD EoS: experimental constraints (2)

S. Pratt et al., Phys. Rev. Lett. 114, 022301 (2015)

Randomly generated (50) EoS for the sound velocity with the following parameterization

(a) Without experimental constraints

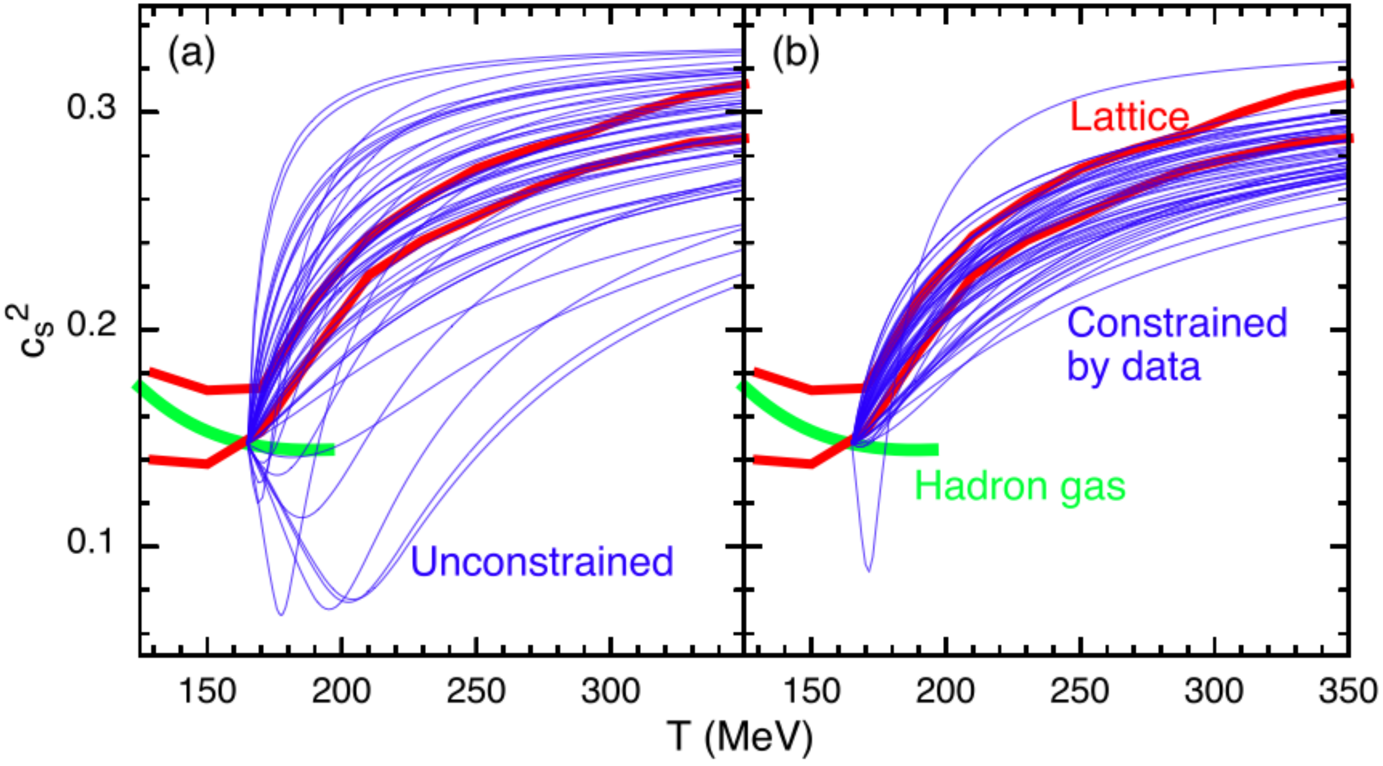
(b) With experimental constraints



$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right)\frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12}, \qquad x \equiv \ln \epsilon/\epsilon_h, \qquad (2)$$

where ϵ_h is the energy density corresponding to $T = 165$ MeV. The two parameters R and X' describe the behavior of the speed of sound at energy densities above ϵ_h. Whereas R describes how the speed of sound rises or falls for small x, X' describes how quickly the speed of sound eventually approaches 1/3 at high temperature. Once



“The resulting constraints suggest the speed of sound gradually rises as a function of temperature from the hadron gas value. The band of equations of state is modestly softer than that of lattice calculation, but has significant overlap.”

Conclusion

➢ Impact of QCD phase transition on cosmology is an open question

- *QGP as the possible source of cosmological dark radiation*, J. Birrell and J. Rafelski, Phys. Lett. B 741 (2015) 77-81
- …

➢ Study of QCD phase transition through hot and dense medium created in heavy-ion collision can produce valuable inputs for Cosmology

➢ New experimental results from LHC (closer to early Universe conditions) need to be taken into account for updated Cosmological implications, via the constrained EoS

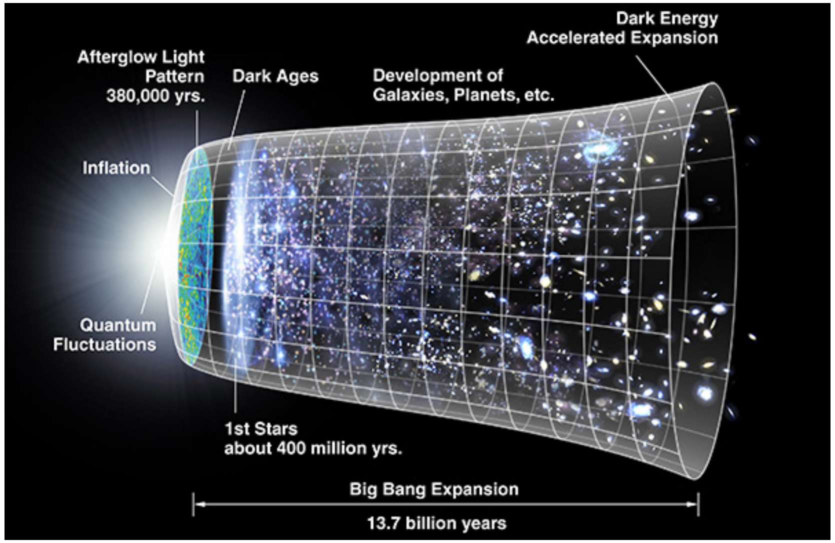
➢ More experimental data (LHC and RHIC) to come in the next decade with new challenging ideas

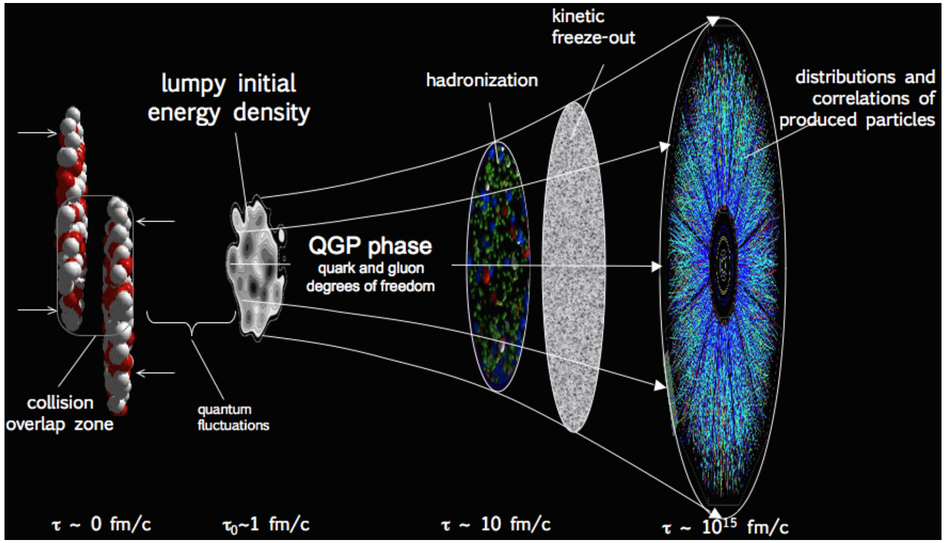
➢ **Interplay between both fields likely to become more important in the next decade** (Stefan Flörchinger, Quark Matter 2015, Kobe, October 1, 2015)

- *Accelerating cosmological expansion from shear and bulk viscosity*, S. Floerchinger, N. Tetradis and U.A. Wiedmann, Phys. Rev. Lett. 114 (2015) 091301
- …

Big bang – little bang: More than an analogy?

Stefan Flörchinger, Quark Matter 2015, Kobe, October 1, 2015





- cosmol. scale: MPc$= 3.1 \times 10^{22}$ m
- Gravity + QED + Dark sector
- one big event

- nuclear scale: fm$= 10^{-15}$ m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- dynamical description as a fluid

- thermal particle bath with initial fluctuations
- space-time evolution of GR

- colliding nucleus with nucleon distribution
- expanding medium in “pre-existing” 3D space

The End

Cosmological relations

Energy density $$\epsilon_{\rm rad} = \frac{E}{V} = \frac{Nhc}{\lambda V} \propto \frac{1}{[a(t)/a_0]^4} = (1+z)^4$$

- $a(t) \propto \sqrt{t}$ = scale factor at time t
- a_0 = scale factor today
- z = redshift corresponding to time t

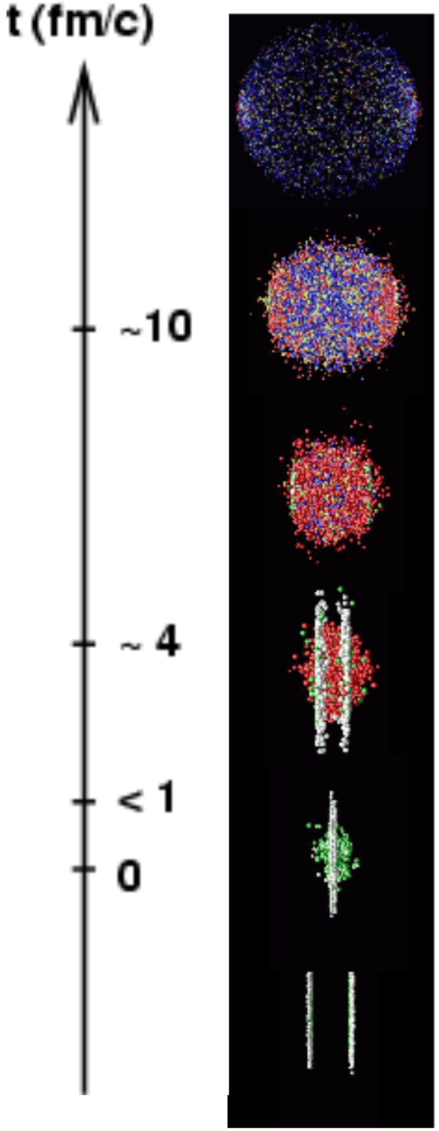
Temperature

$$T(z) = T_0(1+z)$$

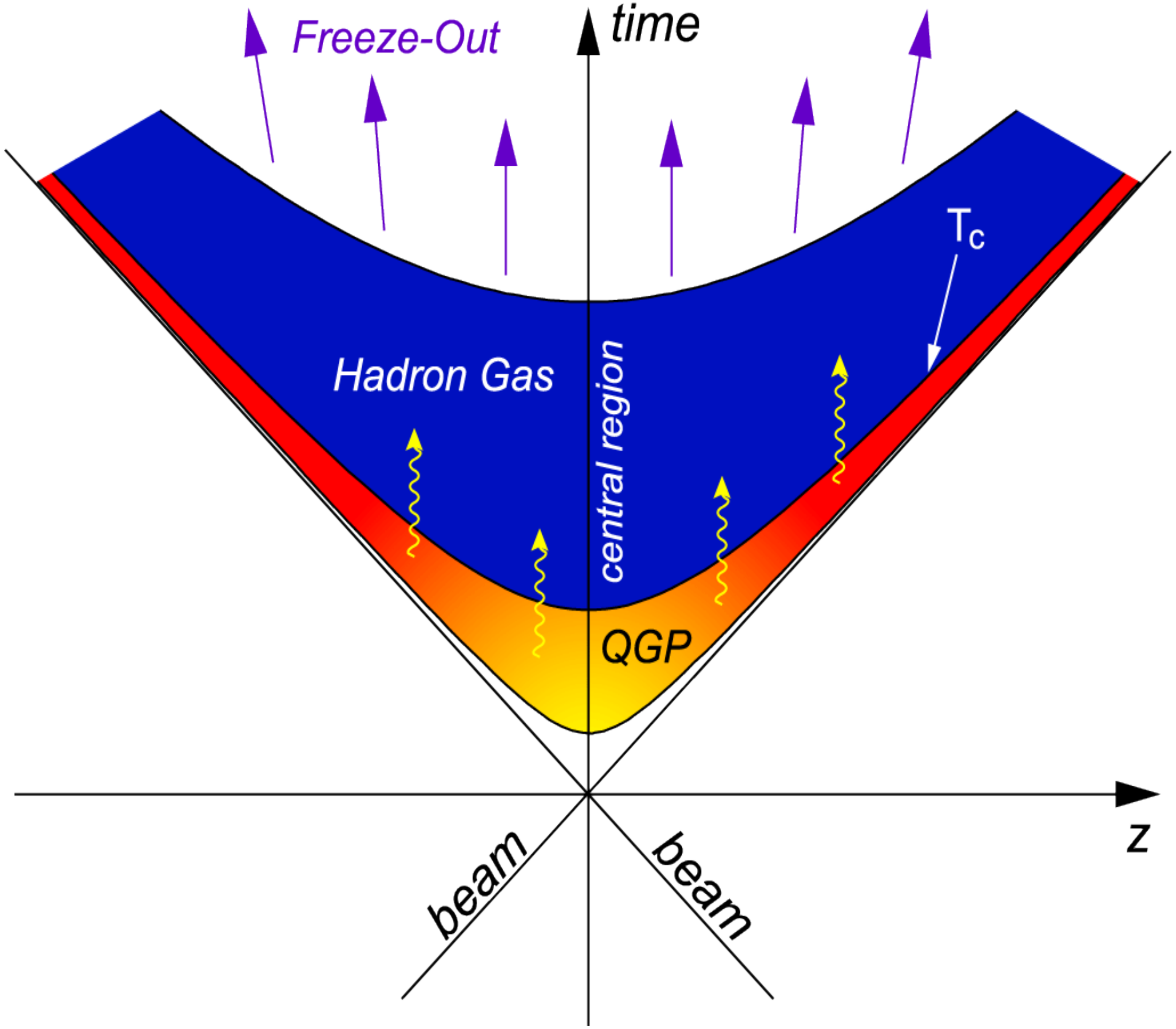
with T_0 today temperature (CMB)

Scale factor $$a(t) \propto \frac{1}{T(t)}$$

Heavy-ion collision picture

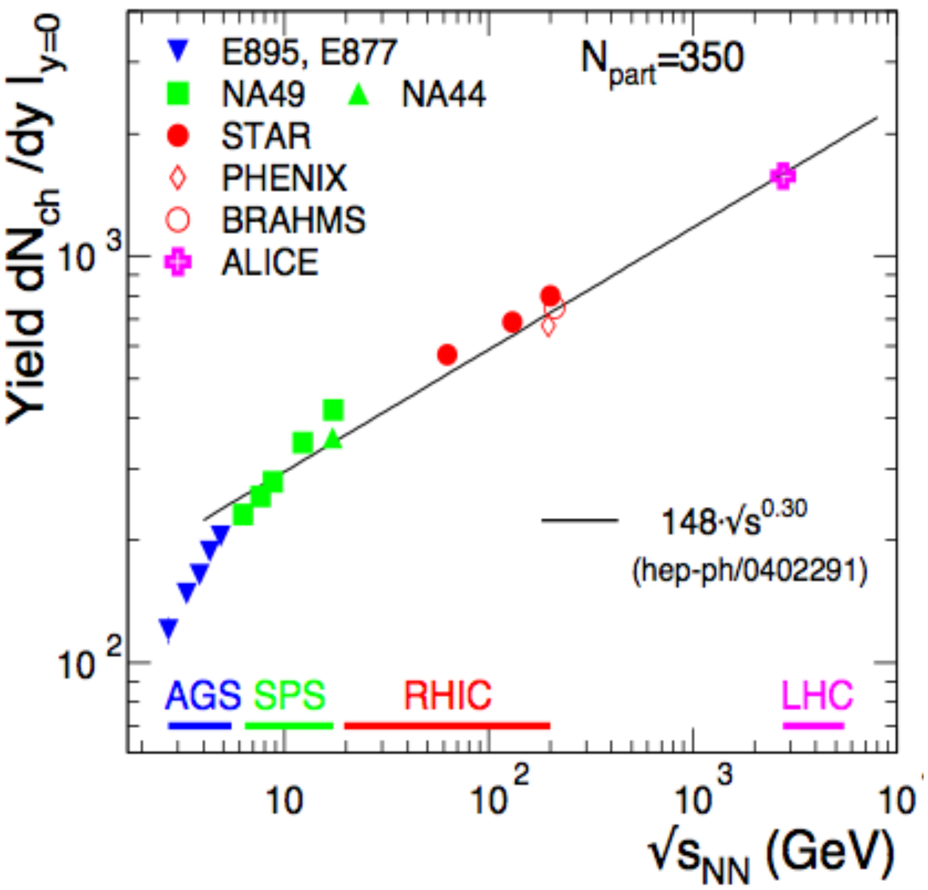


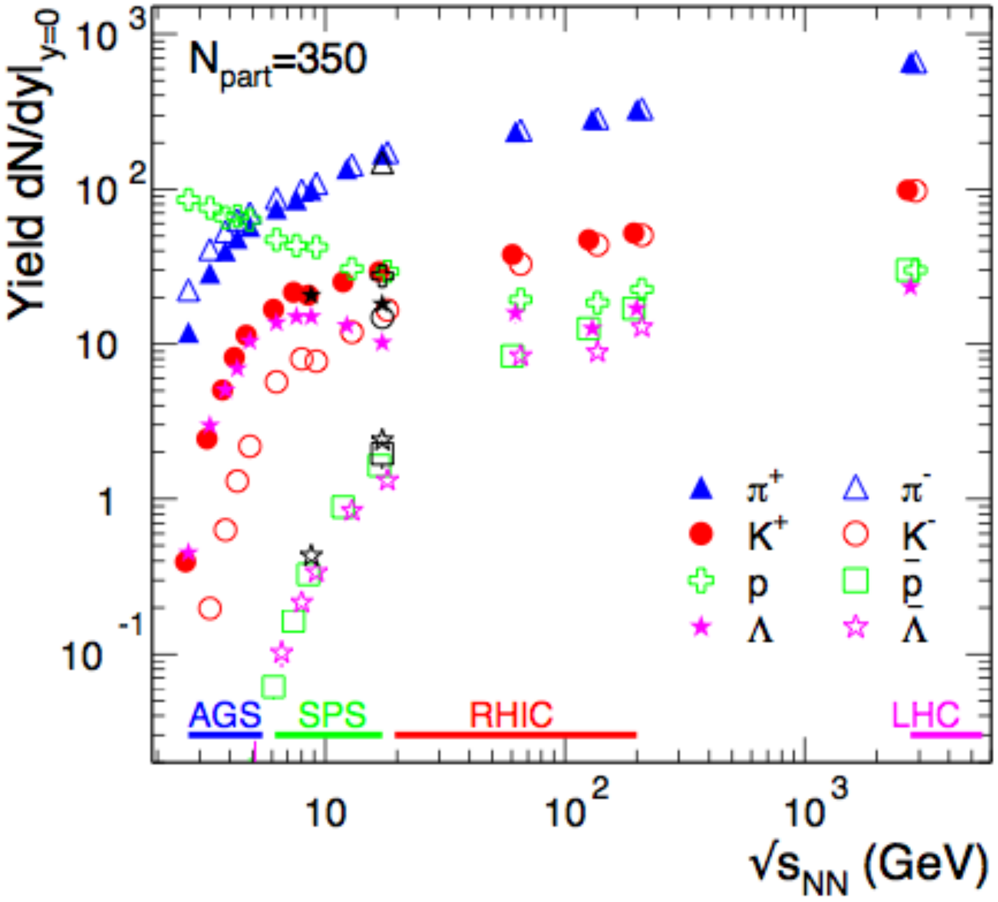
$1\ \text{fm/c} \approx 3 \times 10^{-24}\ \text{s}$



Two nuclei are sent in (frontal) collision at high energy (Lorentz contraction): $d = d_0/\gamma$ with $\gamma = 1468$ at LHC (run 1)

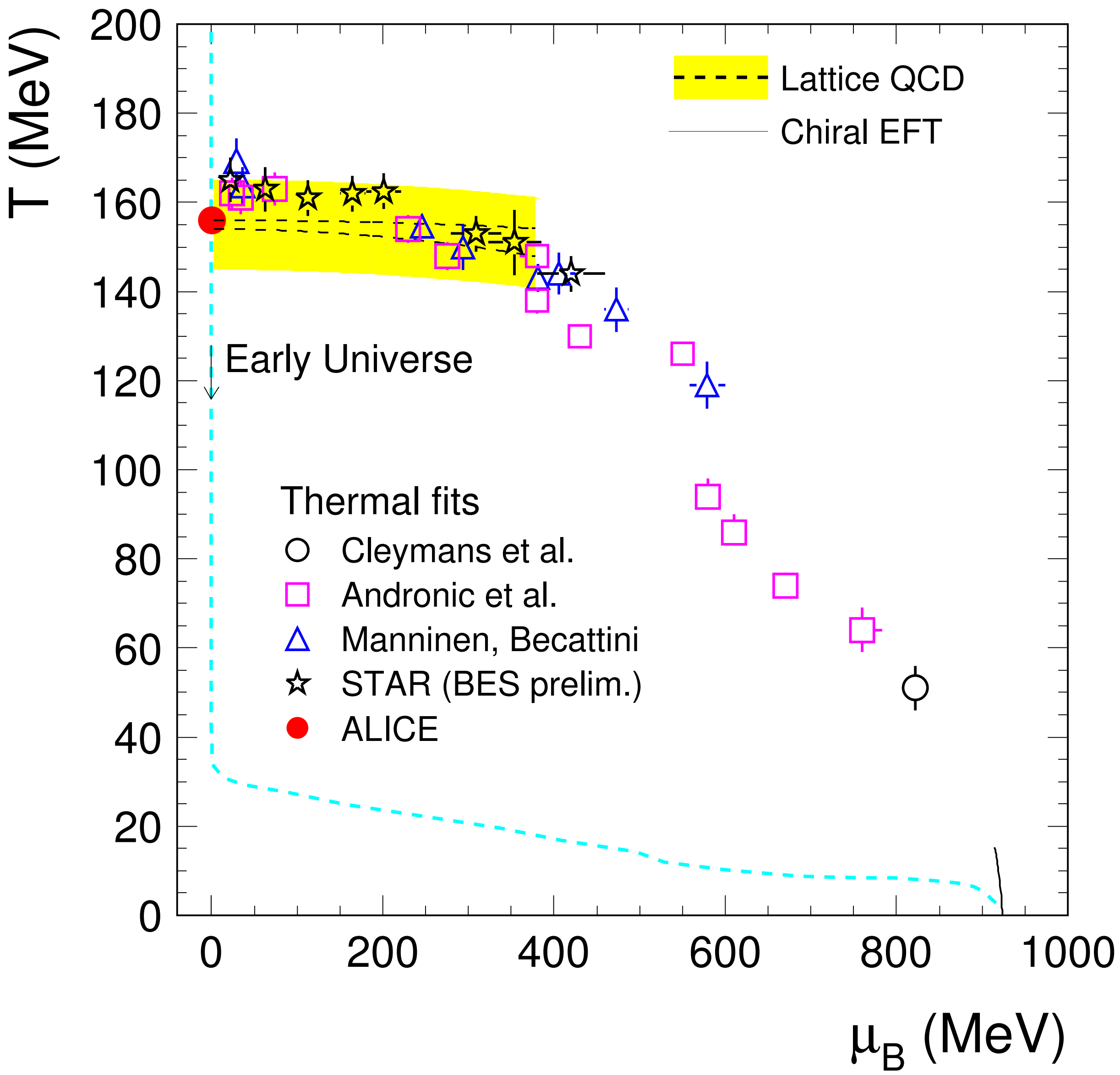
Statistical hadronization in heavy-ion collisions





A. Andronic, Int. J. Mod. Phys. A 2014.29

QCD phase diagram

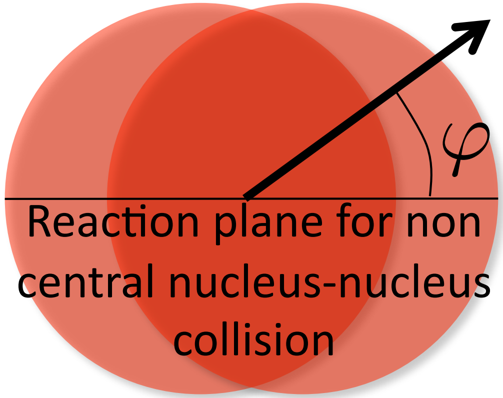


A. Andronic, EMMI workshop: Imprints the QGP, 16-17 April 2015

Hydrodynamic description of heavy-ion collisions

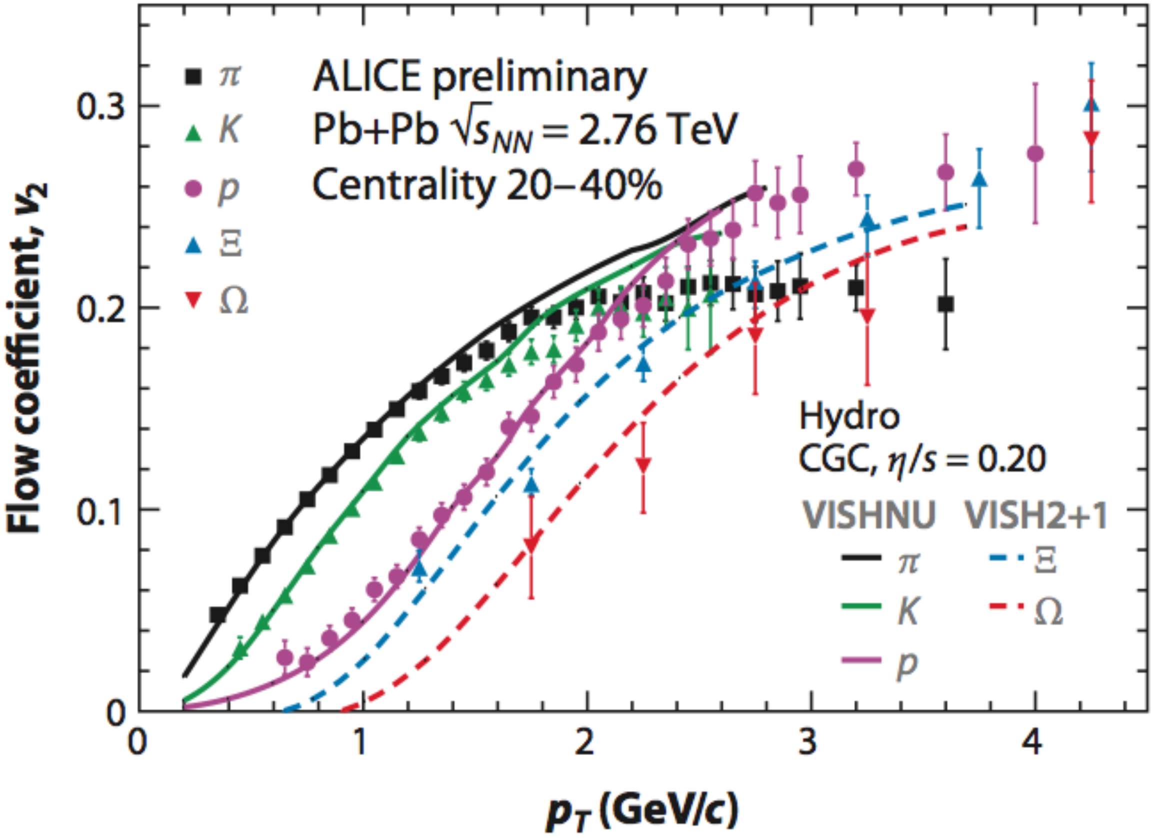
Spatial asymmetry in non central collisions results in anisotropic particle distribution in momentum space (flow)

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n\varphi)\right)$$



Reaction plane for non central nucleus-nucleus collision

- B. Müller, J. Schukraft and B. Wyslouch, Annu. Rev. Nucl. Part. Sci. 2012.62:361-386
- ALICE Collaboration, JHEP 06 (2015) 190



Good description of particle flow by hydrodynamics models

- constraint on QGP EoS
- QGP behaviour in agreement with a nearly perfect fluid (shear viscosity η over entropy density s)

  4πη/s ≤ 2.5

to be compared to the conjectured AdS/CFT limit

4πη/s ≥ 1

P. Kovtun, D. Son and A. Starinets, Phys. Rev. Lett. 94, 111601 (2005)

Hydrodynamic description of QGP

① Energy-momentum conservation (here for a perfect fluid)

$$\partial_\mu T^{\mu\nu} = 0$$

$$\text{with} \quad T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$\epsilon = \text{energy density}$$

$$P = \text{pressure}$$

$$u^\mu(x) = \gamma(x)(1, \mathbf{v}(x))$$

$$g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

② Baryon number conservation

$$\partial_\mu \mathrm{J}^\mu_\mathrm{B} = 0$$

$$\text{with} \quad \mathrm{J}^\mu_\mathrm{B}(x) = n_\mathrm{B}(x) u^\mu(x)$$

$$n_\mathrm{B}(x) = \text{baryon number density}$$

③ Entropy conservation

$$\partial_\mu (s u^\mu) = 0$$

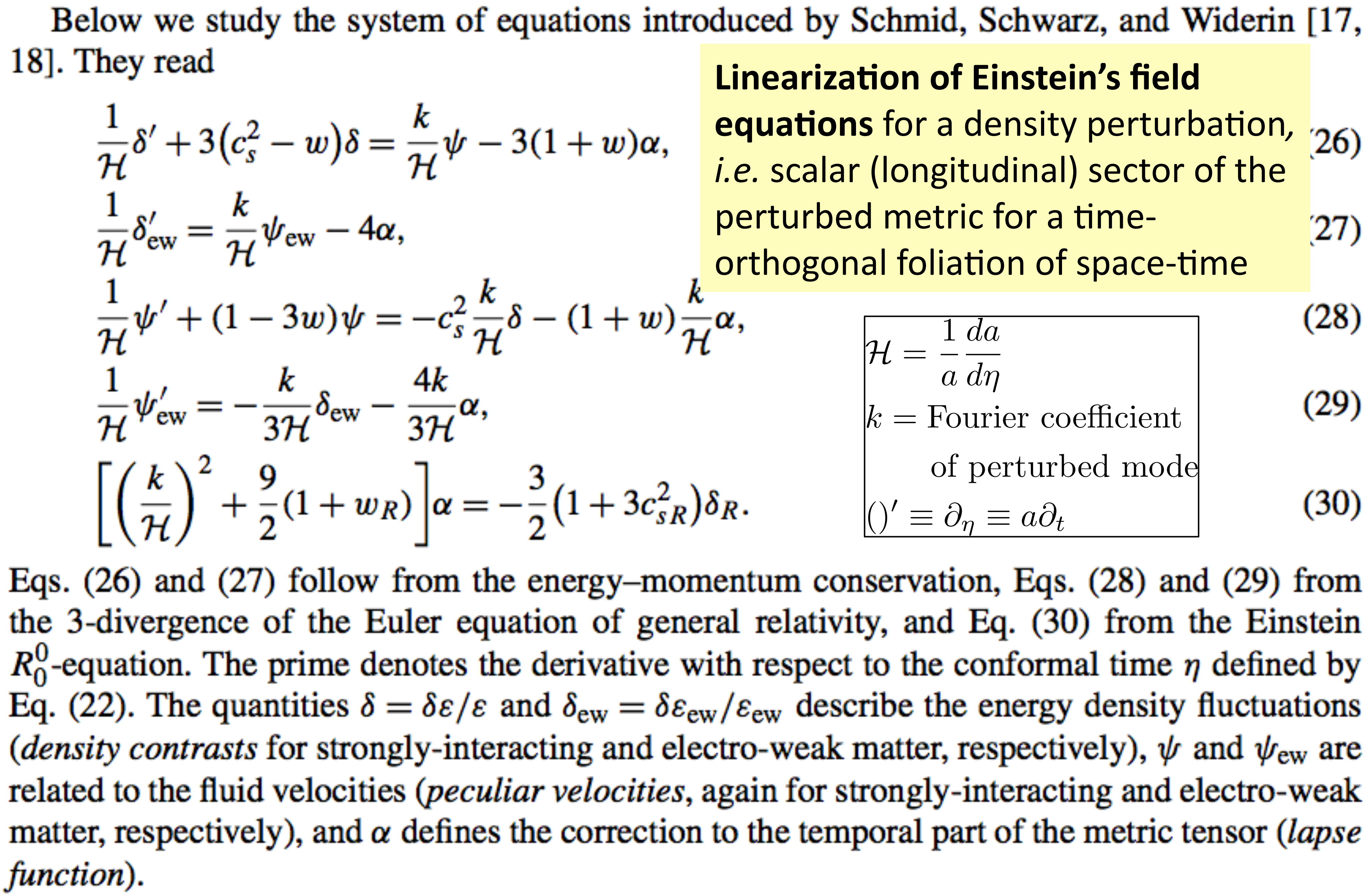
$$\text{with} \quad s(x) = \text{entropy density}$$

④ Equation of state (lattice QCD)

$$p = p(\epsilon)$$

Cosmological inhomogeneities

W. Florkowski, Nucl. Phys. A 853 (2011) 173-188



Below we study the system of equations introduced by Schmid, Schwarz, and Widerin [17, 18]. They read

$$\frac{1}{\mathcal{H}}\delta' + 3\left(c_s^2 - w\right)\delta = \frac{k}{\mathcal{H}}\psi - 3(1+w)\alpha, \tag{26}$$

$$\frac{1}{\mathcal{H}}\delta'_{\mathrm{ew}} = \frac{k}{\mathcal{H}}\psi_{\mathrm{ew}} - 4\alpha, \tag{27}$$

$$\frac{1}{\mathcal{H}}\psi' + (1-3w)\psi = -c_s^2\frac{k}{\mathcal{H}}\delta - (1+w)\frac{k}{\mathcal{H}}\alpha, \tag{28}$$

$$\frac{1}{\mathcal{H}}\psi'_{\mathrm{ew}} = -\frac{k}{3\mathcal{H}}\delta_{\mathrm{ew}} - \frac{4k}{3\mathcal{H}}\alpha, \tag{29}$$

$$\left[\left(\frac{k}{\mathcal{H}}\right)^2 + \frac{9}{2}(1+w_R)\right]\alpha = -\frac{3}{2}\left(1+3c_{sR}^2\right)\delta_R. \tag{30}$$

Linearization of Einstein's field equations for a density perturbation, *i.e.* scalar (longitudinal) sector of the perturbed metric for a time-orthogonal foliation of space-time

$\mathcal{H} = \frac{1}{a}\frac{da}{d\eta}$

$k =$ Fourier coefficient of perturbed mode

$()' \equiv \partial_\eta \equiv a\partial_t$

Eqs. (26) and (27) follow from the energy–momentum conservation, Eqs. (28) and (29) from the 3-divergence of the Euler equation of general relativity, and Eq. (30) from the Einstein R_0^0-equation. The prime denotes the derivative with respect to the conformal time η defined by Eq. (22). The quantities $\delta = \delta\varepsilon/\varepsilon$ and $\delta_{\mathrm{ew}} = \delta\varepsilon_{\mathrm{ew}}/\varepsilon_{\mathrm{ew}}$ describe the energy density fluctuations (*density contrasts* for strongly-interacting and electro-weak matter, respectively), ψ and ψ_{ew} are related to the fluid velocities (*peculiar velocities*, again for strongly-interacting and electro-weak matter, respectively), and α defines the correction to the temporal part of the metric tensor (*lapse function*).